Problem Set 6: Diagonalization and the Cayley-Hamilton Theorem
Due Tuesday, July 26th

From FIS, 3rd edition:

- p. 218, #20 (*Hint:* To calculate the determinant, you can row reduce the top half and the bottom half separately to get an upper triangular matrix).
- p. 248, #3b
- p. 250, #20
- p. 269, #3b
- p. 270, #8
- p. 309, #3b
- p. 311, #18bc (*Hint:* Use the Cayley-Hamilton Theorem), 19
- *Extra Credit:* The purpose of this problem will be to find an explicit formula for the Fibonacci numbers. Recall that the Fibonacci numbers are defined by $F_0 = 0, F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.

  a) Prove by induction that $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} * & F_n \\ * & F_{n+1} \end{pmatrix}$ for $n \geq 1$.

  b) Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$. Diagonalize A by finding a matrix Q such that $Q^{-1}AQ = D$, where D is a diagonal matrix.

  c) Having done this diagonalization, find a formula for A^n.

  d) Combine parts (a) and (c) to find an explicit formula for the nth Fibonacci number.