Problem Set 8: More on Inner Product Spaces
Due Tuesday, August 2nd

- Prove that if A is a real orthogonal matrix, then $\det A = \pm 1$ (Note: this should make sense, because orthogonal matrices preserve length, and thus area—but don't use this for your proof).

From FIS, 3rd edition:

- p. 346, #3b
- p. 347, #13 (Hint: If $T^*Tv = 0$, then $\langle T^*Tv, v\rangle = 0$)
- p. 354, #2 (justify your answers—no need to find the orthonormal basis for part (a)), #3
- p. 369, #2c
- p. 370, #11
- p. 372, #21d
- p. 406, #4acd
- Prove that $H : P_2(\mathbb{R}) \times P_2(\mathbb{R}) \to \mathbb{R}$ given by $H(f, g) = f(3)g(4)$ is bilinear. Write down the matrix $\psi_\beta(H)$, where β is the ordered basis $(1, x, x^2)$.
- *Extra Credit:* p. 356-7, #16. Follow the hint given. Justify carefully why you can assume that A is upper triangular (i.e., don't just say "by Schur's Theorem"). *Warning:* This is a difficult problem to prove correctly—make sure you are rigorous about every step!