Practice Final

1. Let $\langle f, g\rangle$ be defined from $P_2(\mathbb{R}) \times P_2(\mathbb{R}) \to \mathbb{R}$ by $\langle f, g\rangle = f(-1)g(-1) + f(0)g(0) + f(1)g(1)$.

a) Show that this is an inner product.

b) Let $W \subset P_2(\mathbb{R})$ be the subspace of polynomials $\{p \in P_2(\mathbb{R}) | p(1) = 0\}$. Find a basis for $W^\perp$. (*Hint:* W is 2-dimensional, although if you use this fact you should show it).

2. Let $A_\alpha = \begin{pmatrix} 2 & \alpha & 1 \\ 0 & 2 & 0 \\ 0 & 0 & \alpha \end{pmatrix}$, where α is real (so, for instance, $A_1 = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$).

a) In terms of α, find the eigenvalues of A_α, their multiplicities, and the dimension of each of the eigenspaces.

b) For which values of α is A diagonalizable?

3. If a and b are real numbers, compute the determinant of

$$A = \begin{pmatrix} b & a & 0 & 0 \\ 0 & b & a & 0 \\ 0 & 0 & b & a \\ a & 0 & 0 & b \end{pmatrix}$$

in terms of a and b however you wish. When is A invertible?

4. FIS, p.299, #13 (*Hint:* To figure out the eventual percentages, show that A is regular).

5. For the following four functions, state whether they are bilinear forms, inner products, both, or neither. If something is NOT bilinear or NOT an inner product, justify why not (you do not need to justify why something is an inner product or bilinear).

a) $H : \mathbb{C}^n \times \mathbb{C}^n \to \mathbb{C}$ given by the standard complex dot product $H((z_1, \ldots, z_n), (w_1, \ldots, w_n)) = z_1\overline{w_1} + \cdots + z_n\overline{w_n}$.

b) $H : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ given by $H((x_1, y_1, z_1), (x_2, y_2, z_2)) = x_1y_1 + x_2y_2 + 4z_1z_2$.

c) $H : P(\mathbb{R}) \times P(\mathbb{R}) \to \mathbb{R}$ given by $H(f, g) = f(6) + g(7)$.

d) $H : M_{n\times n}(\mathbb{R}) \times M_{n\times n}(\mathbb{R}) \to \mathbb{R}$ given by $H(A, B) =$ sum of the entries in AB.