

Tricky problems for practice

1. The area between the curve $y = x^2$ and the curve $y = x^3 - 6x$ is given by

(a) $\int_{-2}^3 \int_{x^2}^{x^3-6x} 1 \, dy \, dx$

(b) $\int_{-2}^3 \int_{x^3-6x}^{x^2} 1 \, dy \, dx$

(c) $\int_{-2}^0 \int_{x^2}^{x^3-6x} 1 \, dy \, dx + \int_{-2}^0 \int_{x^3-6x}^{x^2} 1 \, dy \, dx$

(d) $\int_{-4\sqrt{2}}^9 \int_0^3 1 \, dx \, dy$

(e) none of the above.

2. The function f is twice continuously differentiable. Which of the following sets of conditions guarantees that the maximum of $f(x, y)$ over the triangle $x + y \leq 1, x \geq 0, y \geq 0$ occurs on the line segment $x + y = 1$?

(a) f obeys the symmetry $f(x, y) = f(y, x)$ and the gradient of f is never zero in the interior of the triangle.

(b) f_x and f_y are both positive.

(c) the gradient of f vanishes at exactly one point in the triangle, namely $(\frac{1}{2}, \frac{1}{2})$ and here $f_{xx}f_{yy} > f_{xy}^2$.

(d) the gradient of f vanishes at exactly one point in the triangle, namely $(\frac{1}{3}, \frac{1}{2})$ and here $f_{xx}f_{yy} < f_{xy}^2$.

(e) none of the above.

3. Which of the following vector functions is NOT the gradient of any differentiable function of two variables on the square $1 \leq x, y \leq 2$?

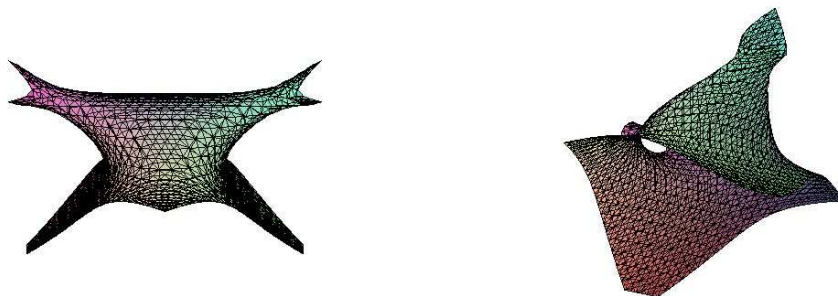
- (a) $f(x, y) = \langle ye^x, xe^x \rangle$
- (b) $f(x, y) = \langle (1/y)e^{x/y}, -(x/y^2)e^{x/y} \rangle$
- (c) $f(x, y) = \langle \sin(1/x^2), (1 + y^2)^{3/2} \rangle$
- (d) $f(x, y) = \langle 3x^2y^5, 5x^3y^4 + 1 \rangle$
- (e) all are gradients of some function

4. Which is the value of the following double integral?

$$\int_a^b \int_0^{\sqrt{\log(b/x)}} \frac{\partial}{\partial y} x e^{y^2} dy dx$$

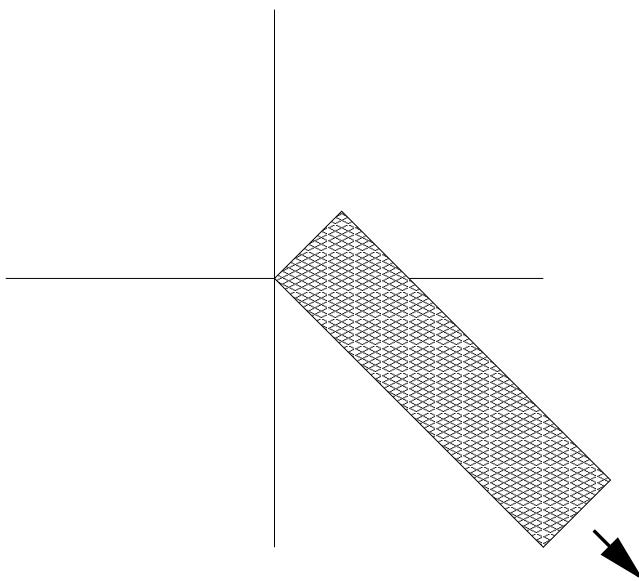
- (a) $b(b - a)$
- (b) $\frac{1}{2}(b - a)^2$
- (c) $b - a$
- (d) $\frac{1}{2}b(b^2 - a^2)$
- (e) the integral is improper and converges to $+\infty$.

5. A Viking battle helmet (two views are shown) has the shape of the surface $x^2 + y^2 + z^3 + xyz = 9$ in the range $0 \leq x, y \leq 5$. A normal to the surface at the point $(1/2, 1/2, 2)$ is given by



- (a) $8\hat{i} + 8\hat{j} + 49\hat{k}$
(b) $49\hat{i} + 49\hat{j} + 8\hat{k}$
(c) $-8\hat{i} - 8\hat{j} + 49\hat{k}$
(b) $-49\hat{i} + -49\hat{j} + 8\hat{k}$
(e) none of the above

6. Let \mathcal{R} be the semi-infinite strip in the picture, bounded by the line segment whose endpoints are $(0,0)$ and $(1,1)$ and by the two rays pointing directly southeast from the endpoints of the segment.



Evaluate the integral:

$$\int \int_{\mathcal{R}} \frac{x+y}{x^2+y^2} dA.$$

7. Suppose $Q(x, y)$ and $M(x, y)$ are two differentiable functions and the triple (x, y, t) solves the following system of equations.

$$Q = 0$$

$$M_x = tQ_x$$

$$M_y = tQ_y$$

Which of these will be true?

- (a) the gradient of M is parallel to the gradient of Q at the point (x, y)
 - (b) (x, y, t) is a critical point for the three variable function $M(x, y) - tQ(x, y)$
 - (c) (x, y) is a critical point for the objective function $M + 1$ on the curve $Q(x, y) = 0$.
 - (d) all of the above
 - (e) none of the above
8. Let S be the sum of $1/\sqrt{1 + m^2 + n^2}$ over all pairs of integers (m, n) such that $m^2 + n^2 < 3000$. Which of the following best approximates S ?
- (a) $\log 3000$
 - (b) $2\pi \operatorname{arcsinh}(3001)$
 - (c) $3000\pi/\sqrt{3001}$
 - (d) $2\pi \log 3000$
 - (e) $2\pi(\sqrt{3001} - 1)$

9. The closest the surface $xy + z = 10$ comes to the origin is a distance of

(a) 3

(b) $\sqrt{10}$

(c) $(\sqrt{123} - \sqrt{3})/2$

(d) $\sqrt{19}$

(e) none of the above

10. An attic room is 12×12 feet and has a sloping ceiling whose height $z = (24 - x - y)/3$ goes from 8 feet at the highest corner down to zero at the opposite corner. A rectangular box is being built in the room. A diagonal line will be drawn on the floor from one corner to the other, and the box must fit entirely on one side of this line. What is the greatest possible volume of such a box?