Word Problems and Qualitative Conceptual Problems

1. A plane can fly at 180 km/h in still air. The plane attempts to head North, but after 30 minutes the plane has actually travelled 80 km at an angle 5° East of North. What is the wind velocity and in what direction should the pilot have headed to reach the intended destination?

2. Describe the set of points Q such that $\vec{PQ} \times \mathbf{B} = \vec{0}$.

3. A ball rolls off a table with a speed of 2 feet per second. The table is 3.5 feet high. At what angle does the ball hit the floor?

4. How fast do you have to hit a baseball to clear a 30 foot fence that's 400 feet away, assuming the baseball is 2 feet off the ground when you hit it (ignore air resistance)?

5. A *transfer curve* between two straight segments of train track is a connecting curve which makes the slope continuous and the curvature continuous (so there is no sudden change of acceleration). Find a polynomial $y = P(x)$ transfer curve between the negative x-axis and the digaonal $y = x$, for $x \geq 1$.

6. A bug crawls at speed 1 inch per second out from the center of a merry go round spinning at 1 revolution per second. At what point is the bug's normal acceleration equal to its tangential acceleration?

7. Which of these functions, expressed in polar coordinates, are continuous at the origin? (evaluate f at each pont by choosing a positive r and a θ in the interval $[0, 2\pi)$)

$$\begin{aligned} f(r,\theta) &= \theta \\ f(r,\theta) &= r \\ f(r,\theta) &= r\theta \\ f(r,\theta) &= r\sin(\theta) \\ f(r,\theta) &= r\tan(\theta) \end{aligned}$$

8. The *kinetic energy* of an object is one half the mass times the square of the speed. If an accelerating object gains mass but keeps the same kinetic energy, then what is the rate of change of mass with respect to speed?

9. Suppose the temperature at a point in the plane is given by a function $f(x, y)$, where x and y are the Cartesian coordinates. In polar coordinates, what is the rate of change of temperature with respect to r when θ is held fixed?

10. Three resistors R_1, R_2 and R_3 are in parallel. Someone changes the first resistance, and your job is to change the second so as to compensate and keep the total resistance constant. What should be the rate of change of the first resistor with respect to the second?

11. Two lab workers are trying to compute the mixed partial derivative $\partial^2 f/\partial x \partial y$ of temperature with respect to concentrations of two ingredients. Here are their measurements of temperature at various concentrations. As you can see, Lab Worker #1 measured temparature to two deccimal places and used concentrations differing in one decimal place, while Lab Worker #2 measured temperature to three decimal places and used concentrations differing in the second decimal place.

    Assume that f has continuous derivatives that don't vary wildly.

    (a) What did the first lab worker's data indicate about the true value of the mixed partial?

    (b) What did the second lab worker's data indicate?

    (c) Which estimate do you trust more and why?

	$x = 1.0$	$x = 1.1$
$y = 1.0$	3.67	3.98
$y = 1.1$	3.36	3.70
$y = 1.2$	3.00	3.38

	$x = 1.00$	$x = 1.01$
$y = 1.00$	3.675	3.708
$y = 1.01$	3.646	3.680
$y = 1.02$	3.680	3.651

12. If I drive South, it gets warmer at a rate of 1 degree every 25 miles. If I drive West, it gets colder at a rate of 1 degree every 75 miles. Can you tell from this at what rate the temperature changes when I drive Southwest?

13. If I drive South, it gets warmer at a rate of 1 degree every 25 miles. If I drive West, it gets colder at a rate of 1 degree every 75 miles. Which direction should I drive to get warm the fastest?

14. The function $(x-1)^2 + y^2 + 1/(1 + x^2 + y^2)$ has only one critical point. Can you tell without too much computation whether it is a local maximum, a local minimum or a saddle?

15. What is the closest point to the origin on an ellipse centered at the point $(5, 2)$ with width 2 and height 1 (parallel to the coordinate axes)?

16. Why doesn't the method of Lagrange multipliers seem to work at first to solve this problem, and how can you fix it? Minimize $x^2 + x + y^2$ subject to $x^2 - 2xy + y^2 = 0$.

17. Let $F(x, y)$ be the total number of shrubs in a rectangle from the origin to (x, y) in a patch of land where the density, $\rho(x, y)$ of shrubs does not depend on x. What is $\partial F/\partial y$?

18. The quarter of the unit disk that is in the first quadrant has varying density proportional to the x-coordinate. Where is its center of mass?

19. The quarter of the unit disk that is in the first quadrant has varying density proportional to the distance from the origin. Where is its center of mass?

20. What is the average height above the floor in a dome with hemispherical ceiling, as a fraction of the maximum height?

21. An explosion ionizes dust particles, with the number of particles per cubic meter ionized at distance r from the explosion given by $10^5 e^{-r/40}$. How many ionized particles will be produced altogether?

22. Suppose air resistance is proportional to the square of the velocity instead of the first power. Write an ODE for this and solve it. Assume the only force is air resistance (no gravity).

23. The rate of change of the price, P, of a pound of coffee is given by $(0.01)(\$15\,t - P)$. If the price at $t = 1$ is \$10, what is the price at $t = 10$ and at $t = 1000$?

24. Suppose the rate of warming of a pond in degrees per day is equal to the difference in temperature between the pond and the air, and that the air temperature in degrees Celsius is $5\sin(2\pi t)$ where t is measured in whole days and $t = 0$ is dawn on December 1. What, roughly, is the temperature of the pond at dawn on December 15? Roughly what time of day will be the warmest during most of December?

25. A population grows in time, with the net growth equal to birth rate minus death rate, where birth rate is proportional to the existing population and death rate is proportional to the square of the population. Write a differential equation for this and solve it. What happens as time goes to infinity?

26. A yam in an oven at 400° has temperature satifying $T' = k(400^\circ - T)$ degrees per hour. If the yam starts at room temperature (300°) and is found 30 minutes later at 350°, what is the constant k?

27. A string of lenght L hangs from $(0, L)$, so that a weight attached to the bottom is at the origin. Suppose it swings in the x-y plane so that the angle it deviates from hanging straight down is $\theta(t)$. Conservation of energy dictates that the speed should be $\sqrt{2g(y_{\max} - y(t))}$, where $y_{\max}$ is the maximum height to which it will swing. Write a differential equation for θ.

28. Suppose a tired jogger keeps reducing her speed so it is proportional to the distance she has remaining to go. Write an ODE for position as a function of time and solve it to find out how close she is to the finish at time t, as a function of any constants you need to introduce.