Homework II: Stat 530 version

For the modeling situations in Prblems 1–3 below, construct a probability model. You may define the space Ω explicitly or you may say, "Let Ω be a space on which are defined the following random variables with the following properties." You must define a formal random variable for each real-world quantity you discuss, and you must state this correspondence.

1. Construct a probability model for the game of Blackjack, and define the event that the dealer will bust. If you do not already know the rules of Blackjack, you will need to find them out; assume here that the dealer completes his hand even if all palyers have busted. Blackjack is usually played with a super-deck obtained by shuffling together many regular decks of cards; you may use a single deck in your model, or you may use an "infinite-deck" approximation (the infinite deck probably yields better answers). You do not need to compute the probability of the dealer busting, but you might want to do this for fun.

2. People concerned about Global warning have cited the large number of recent record high temperature years as a source of concern. A possible counter-argument is that these records could occur by chance. Construct the simplest chance model you can for the data, the data being a single temperature for each year from 1925 to 2004 representing the worldwide average temperature for that year. How many record-high years would you expect in this model? What is the probability under this model of having a record high temperature in both 2003 and 2004? (You should give a number but you don't have to prove it.)

3. Continue the last problem using a more sophisticated model in which the default assumption is not that the temperatures in successive years are unrelated, but that the present year's temperature is likely to be a good approximation for next year's temperature, with a small, and unpredictable change (but no global warming trend is assumed). Is the probability in this model for record highs in 2003 and 2004 higher or lower than in the previous model (you don't need to prove your answer or to compute a number for the probability)?

4. The random number generator in a popular 2001 Spreadsheet seems to have the property the first and second numbers it generates will have the right joint distribution, and the same holds for any fixed pair, but that if you plot points in three-space that are sample values of a triple such as the first three numbers, the distribution appears very non-uniform. Model this by defining a probability space with random variables which you state to satisfy certain hypotheses. Please say how the properties hypothesized relate to the properties stated in the text of the problem. Then give an example to show such a probability space exists. Finally, formulate a theorem about the average of the first n numbers generated.

5. Suppose $X_1, \ldots, X_n$ are IID standard normals and $\{a_{ij} : 1 \leq i \leq k, 1 \leq j \leq n\}$ are real numbers. Under what conditions are the variables $Y_i := \sum_j a_{ij} X_j$ jointly independent as i runs from 1 to k? (To answer this you will probably need to compute the joint density of $(Y_1, \ldots, Y_k)$ using the change of variables formula.)