THE EFFECTS OF THE USE OF PROBLEM SOLVING AND COOPERATIVE LEARNING ON THE MATHEMATICS ACHIEVEMENT OF UNDERPREPARED COLLEGE FRESHMEN

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ABSTRACT: The study involved first semester freshmen (graduated from high school the summer immediately preceding their enrollment in MA 101 (Algebra)) enrolled in MA 101 and registered for MA 111 (Precalculus) the semester after completing MA 101. The researcher gathered the necessary background data (high school grade point average (HSGPA), gender, SAT-M, SAT-V, MA 101 grade, and MA 111 grade) on all students involved in the study. It should be noted that MA 101 does not satisfy a student's mathematics requirement at the university, however MA 111 does. It is customary for a student to take MA 101 and MA 111 in successive semesters.

Specifically the objectives of the investigation were:

1. To compare instructional methods as measured by the end of course grade of the next mathematics course in order to determine whether the relationship between the course grades for MA 101 and MA 111 was different in the 1988-1989 and 1989-1990 school years.
2. To investigate whether MA 111 grades, on average, were higher in 1990 than 1989 in order to determine whether the instructional intervention was successful in helping students understand, retain, and transfer their knowledge of algebra.

MA 101 instruction during the fall semester of 1988 (control group) was of the "standard" lecture variety. During the fall of 1989 (treatment group), problem solving through cooperative learning was emphasized. MA 111 was taught using "traditional" lecture consisting of "chalk and talk" and "paper/pencil" teaching during both years of the study to both the control and the treatment group.

A *significant* difference between performance in MA 111 from one year to the next was found (five years of grades were evaluated). The MA 101 grade was found to be a *significant* factor in the performance in MA 111. The MA 101 grades for students in the control group were *significantly* higher than those of the treatment group. Conversely, the MA 111 grades in the treatment group were significantly higher than those of the control group. Of the students in the control group, 46% successfully completed MA 111, whereas, 70% of the students in the treatment successfully completed MA 111. Thus, the use of cooperative learning and problem solving is an effective technique for improving mathematics achievement with this group of students.

KEYWORDS: Cooperative learning, problem-solving.

Research involving the use of cooperative learning at the college level has found that students who used cooperative learning performed as well as students in more traditional classes and often developed a more positive attitude toward each other and toward mathematics [2, 22, 6]. Some of the studies have found the greatest benefit of the use of cooperative learning is in learning concepts and problem solving [4, 33].

Research has indicated that mathematics achievement in college is of particular importance to the persistence and success of at-risk students. Waits and Demana [32] state that there is a strong relationship between mathematics skills and success in college, regardless of major. They report that only 28% of the remedial college mathematics students in their study reached senior status by the end of four years. In contrast, 81% of students who placed into calculus had reached senior status or had graduated at the end of four years [32].

At North Carolina State University, MA 101 is designed to serve at-risk or underprepared college freshmen who lack the requisite skills or test scores to enroll in mathematics courses that will apply toward graduation. MA 101 covers the following topics: algebra and problem solving; functions, graphing, linear equations; systems of equations; inequalities and linear programming; polynomials and polynomial functions; rational expressions, equations, functions; exponential and radical symbolism; complex numbers; and quadratic equations and functions. Many colleges and universities across the nation offer developmental mathematics courses similar to Math 101 at North Carolina State University [15, 3, 34].

The need to offer developmental mathematics courses is due in part to the type of instruction that these students have received in the past [24]. Many students have difficulty learning mathematics conceptually through lecture and drill-and-practice, and therefore, rely on memorization to get them through their mathematics courses. Memorization without understanding limits the students' ability to perform effectively in mathematics. The lecture/drill-and-practice approach runs counter to the constructivist view of learning mathematics which holds that learning is a personal matter and that real understanding involves the internalization of concepts and relationships by the individual [21]. Research by mathematics educators suggest that students retain best the mathematics that they learn by the process of internal construction and experience [18].

To help students in this process of constructing their own knowledge, many mathematics teachers are adopting new strategies for teaching mathematics. The two strategies most often used are the problem solving inquiry approach and cooperative learning [1].

Problem solving was intended by the NCTM to be the focus of mathematics instruction in the 80's [18], and has since remained one of the more enduring interests of mathematics educators. Problem solving is defined as a process by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. The student must synthesize what she or he has learned and apply it to new and different situations [14]. Problem solving is considered to be the link between facts and algorithms and the real-life problem situations that we all face [14]. The rapid advancements in instructional technology make it more difficult to predict what kinds of mathematics will be required of students for the twenty-first century. One topic, however, is likely to remain constant, and that is problem solving. Problem solving is one of the basic skills students use throughout their lives.

Problem solving involves students in the actions of asking questions, seeking information, and doing an investigation. The goals of a problem -centered curriculum are to teach students to be self-directed, critical thinkers, and problem solvers. The teacher takes on the role of "guide-stimulator" - posing problems, asking open-ended questions, assisting when needed, but also encouraging independence. When using problem solving, the process of learning is at least as important as the *product* [1].

Cooperative learning is a set of instructional strategies which bring students of all performance levels together to work in small, mixed-ability learning groups. The groups vary in size from two to four members for most problem solving experiences. The students in these groups are not only responsible for learning the material being taught in class, but also for helping their group members learn the material. Often, there is a group objective or goal involved [28, 10].

Small-group cooperative learning provides an alternative to both traditional whole-class expository instruction and individual instruction systems. It involves more than just putting students together in small groups and giving them a task; it also involves very careful thought and attention to various aspects of the group process. Cooperative learning classrooms are intended to maximize the active participation of each student and reduce the isolation of individuals. A setting that values and promotes social interaction provides students with an ingredient essential for learning [6].

There are many advantages to learning mathematics in cooperative groups [6].

1. Students learn mathematics by working at a comfortable pace.

2. They learn to cooperate with others and to communicate in the language of mathematics.

3. The classroom atmosphere tends to be relaxed and informal, help is readily available, questions are freely asked and answered, and even the shy student finds it easy to be involved.

4. Students tend to become friends with their group members, and the teacher-student relationship tends to be more relaxed, more pleasant, and closer than in a traditional approach.

5. The usual "discipline" problems of talking and moving around are eliminated by definition.

6. Many students maintain a high level of interest in the mathematical activities.

7. Many students like mathematics more - or at least hate it less.

8. Students have an opportunity to pursue the more challenging and creative aspects of mathematics, while they achieve at least as much information or skill as in more traditional approaches.

According to Slavin [29], two conditions are essential if the achievement effects of cooperative learning are to be realized. First cooperative groups must have group goals that are desirable to the students. The second condition is individual accountability. Success of the group is based on the individual learning of all members. This will not only motivate the maximum learning efforts of all group members, but will elicit encouragement and help among group members that is directed at increasing their learning.

Like Slavin, Johnson and Johnson [12, 11] stress the importance of individual accountability. However, they also emphasize four other basic elements of the Learning Together approach to cooperative learning: positive interdependence, face-to-face promotive interaction, social skills, and group processing. Students must understand and believe that they are responsible for the learning of other members in their group as well as their own (positive interdependence). They must also be given the opportunity to communicate their understanding to other group members in order to complete the group's assignment and to assure that others understand it (face-to-face promotive interaction). Students must also develop trust among group members, settle disagreements, and periodically evaluate how well the group is working together (social skills and group processing). According to the Johnsons, these five basic elements must be included within each cooperative lesson.

Slavin [26] reviewed 46 cooperative learning studies conducted in regular classrooms. Of these, 63% showed cooperative learning methods to have a significantly positive effect on achievement; 33% were found to have no significant effect on achievement when compared with a traditional individual approach; and 4% found a significantly higher achievement for a control group than for a cooperative treatment. In addition, out of 25 of these investigations, 88% found positive effects on student achievement when group rewards were used. In contrast, of the studies that did not use groups rewards, none (of the 9) found positive achievement results. Of the 17 studies that specifically involved mathematics, 10 (59%) found cooperative learning methods to have a significantly positive effect on achievement; 6 (35%) found no significant effect; and 1 (6%) found significantly higher achievement for the control group.

Researchers have found that studies involving the use of cooperative learning at the college level generally have found that students performed just as well as students in more traditional classes, and often developed improved attitudes toward each other and mathematics. Davidson [5, 7] reviewed about 80 studies in mathematics, comparing student achievement in cooperative learning versus whole-class traditional instruction. In over 40% of these studies, the students in the cooperative learning groups significantly outscored the control students on individual mathematical performance measures. In only 2% of the studies did the control group outperform the cooperative learning group.

Recent research concerning cooperative learning has indicated that team rewards and individual accountability are essential elements for producing basic skills achievement [26, 30, 19]. However, Davidson [5] questioned the need for these elements at the college level. Davidson cited examples of successful use of cooperative learning at this level without group goals or individual accountability.

Dees [8] conducted a study to determine whether cooperative learning would help students increase their problem-solving skills in mathematics. The study involved 100 underprepared students registered for a semester course in algebra and geometry. Students using cooperative learning performed as well as or better than the control group on every measure and showed significant differences in favor of cooperative learning when solving word problems in algebra and proof-writing in geometry. Dees concluded that the use of cooperative learning as an instructional intervention at the college level was justified by the increase in the students' problem-solving ability.

Duran and Cherrington [9] conducted a study to examine the effects of cooperative groups versus independent practice in problem-solving lessons. The researchers found significant differences in students' long-term retention of problem-solving strategies. The researchers stated that student in the cooperative groups showed better long-term retention of problem-solving strategies than those who used independent practice. Other researchers [30, 7] that the use of cooperative learning in the mathematics classroom can result in positive influences in problem-solving skills, mathematics achievement, and attitudes toward mathematics.

Problem solving in mathematics is one of the areas where students can benefit from cooperative learning. Placing students in cooperative groups is an effective way to stimulate cognitive development, increase self-esteem, and raise achievement [5]. When students work cooperatively to solve problems, their dialogue helps solidify their thinking and promotes positive attitudes and confidence. Students' learning is supported when they have opportunities to describe their own ideas, hear others explain their thoughts, speculate questions, and explore various approaches. Learning together in small groups gives students more opportunities to interact with concepts than do class discussions. Not only do students have the chance to speak more often, but they may be more comfortable taking the risk of trying out their thinking during problem-solving situations in the setting of a small group.

PURPOSE

The purpose of this study was to compare a problem solving/cooperative learning approach that engaged developmental college algebra students in active learning, with traditional lecture teaching in MA 101 (Algebra). Underprepared students were monitored through one year of college mathematics to determine the effects of these two instructional approaches on their success in their next college mathematics course, MA 111 (Precalculus). Although MA 111 was taught using a lecture format, it was the belief of these researchers that by using cooperative learning and problem solving techniques in the previous course, MA 101, that the students would be able to retain the material taught in MA 101 and be able to transfer this knowledge to new situations, MA 111.

MA 101 is a developmental algebra course offered without credit toward graduation for students who fail to make an acceptable grade on an algebra placement test given during orientation. The subjects were first semester freshmen (graduated from high school the previous June) enrolled in MA 101 during the Fall semesters of 1988 and 1989 who registered for MA 111 the semester immediately following their successful completion of MA 101.

Specifically the objectives were:

1. To compare instructional methods as measured by the final course grade of the next mathematics course in order to determine whether the relationship between the course grades for MA 101 (Algebra) and MA 111 (Precalculus) was different in the 1988-1989 (control) and 1989-1990 (treatment) school years.

2. To investigate whether MA 111 grades, on average, were higher in 1990 than 1989 in order to determine whether the instructional intervention in was successful in helping students understand, retain, and transfer their knowledge of algebra.

PROCEDURE

Table of Sample Population

	1988-89	1989-90
MA 101	212	178
MA 111	212	178

Setting

The study was conducted at North Carolina State University during the Fall and Spring semesters of 1988-1990. This university serves approximately 27,000 students. In 1988-1989 school year (called 89 henceforth), 212 freshmen were enrolled in MA 101, received passing grades, and went on to take MA 111 in the spring. MA 101 instruction in this school year was of the "standard" lecture variety. In 1989-1990 school year (called 90 henceforth), 178 freshmen were enrolled in MA 101 classes emphasizing problem solving through cooperative learning. In each school year, there were several sections of each course taught by several different instructors. All of the instructors involved in the study were either graduate students in mathematics or mathematics education or mathematics instructors. Available data are final grade in MA 101 (5=A, 4=B, etc.), final grade in MA 111 (5=A, etc.), section and instructor for each class, gender, HSGPA, Math and Verbal SAT scores, and mathematics pretest score taken during freshmen orientation.

Sample

All of the students who participated in the study had successfully completed Algebra I, Algebra II, and Geometry in high school. However when these students took the mathematics placement test for North Carolina State University, they scored below the cut-off scores which would have enabled them to take MA 111 (Precalculus) as first semester freshmen. Instead, these students were placed in MA 101 (Algebra), which does not apply toward mathematics credit needed for.graduation.

The study involved 15 sections (control group, $n_1 = 212$) of MA 101 during the Fall of 1988, and 14 sections (treatment group, $n_2 = 178$) of the same course during the Fall of 1989. The students from these sections who registered for precalculus the following semester, were then followed through MA 111.

Instructor Training

The training in cooperative learning of the MA 101 instructors who taught the treatment groups was accomplished by supplying the instructors articles about the use of cooperative learning in the mathematics classroom, watching and evaluating videos of classes using cooperative learning, and receiving feedback from observations by the researcher of these instructors of mathematics classrooms where cooperative learning was being used. To attempt to assure uniformity, the researcher monitored the use of cooperative learning and problem solving by making unannounced visits approximately every two weeks to the instructors' classrooms to observe the entire class period, and teaching some of the teaching assistants' classes by modeling the use of cooperative learning in a problem solving situation.

The following guidelines were given to the instructors to facilitate management of the cooperative learning approach.

- It was the responsibility of the instructor to form the groups and to initiate the group work.
- In addition the instructor was to prepare and introduce the materials for the lesson.
- As the groups were working, the instructor was to interact with the groups by checking solutions, and answering some questions by giving hints and asking thought-provoking questions.
- It is important that the instructor strike a balance between giving too much and giving too little assistance to the groups [6].
- Another responsibility of the instructor was to help the groups tie their ideas together and evaluate each student's performance.
- Lastly, the instructor assigned follow-up work for the students to complete in and out of class.

All of the instructors for MA 101, for both treatment and control groups, were graduate assistants with a Bachelor of Science degree in mathematics education. These students were trained mathematics teachers, had experience teaching mathematics in the public school, and were quite familiar with the problem solving inquiry approach and the use of cooperative learning in the mathematics classroom.

The instructors for MA 111, however, were not mathematics educators. All of these instructors were teaching assistants (graduate students) with a Bachelor of Science degrees in mathematics, and had had little or no formal experience or training in the teaching mathematics. Both MA 101 and MA 111 instructors were trained in grading tests, and the first tests were graded in groups to insure uniformity in grading. Partial credit was given by all instructors, and the amount of credit to be given was discussed during the group grading.

TREATMENTS

Development of Instructional Materials

The researchers reviewed several intermediate algebra books to compile a list of problems for use in the classes. These problems were then solved by three of the instructors, and their comments were reviewed. Problems that were deemed appropriate, according to Polya's [20] definition of problem solving, were then selected.

Organization of the Treatment

All classes met Monday through Friday for 50 minutes a day. The students in the control group were taught algebra by the traditional "lecture" method, while the students in the treatment group were actively involved in both cooperative learning and problem solving as part of their daily instruction. The problems assigned in the treatment group sections of MA 101 were selected to promote higher-order thinking and to provide experiences that go beyond the exercises typically included in textbooks. After the lessons were presented, the students in the treatment group were then instructed to spend the rest of the class period working on assigned problems and answering questions posed by the other group members. These problems were taken from the textbook. One class per week was used entirely for cooperative learning and problem solving. Each group was given a worksheet with 5-8 problems that were to be completed as a group. At the end of the period one member of each group was responsible for presenting the solution to one of the problems to the class. The groups did not know in advance which problem they would be responsible for presenting nor did they know who from their group would be required to present the solution. Therefore, each group had to work all of the problems and ensure that each group member would be able to explain each of the solutions. The group received a single grade based on the group product. In this study, the group grade was used as a reward to motivate and improve the students individual grades.

The MA 101 students in the control group were assigned problems to work individually after the lecture was completed. The problems were taken from the textbook.

Although both treatment and control groups covered the same content, different textbooks were used. The treatment group used a textbook which focused more on the use of problem solving and solving word problems. The control group used a text which stressed algorithms and had a few chapters devoted to solving word problems by type.

Both the treatment and the control groups received the same type of "lecture" instruction in MA 111. Many of the same instructors were used for both the treatment and the control groups. All classes were monitored by the coordinator of MA 111. The same materials and books were used for both groups of students.

Problem Solving Using Cooperative Learning

The students in the treatment sections were grouped in fours at least once a week to work cooperatively on problem solving. The students were heterogeneously grouped according to ability as measured by test and quiz grades. Each group contained one high ability student, one low ability student, and two average ability students. Additionally, the students were encouraged by their instructors to use their groups when studying for tests and in doing assignments in order to provide a support network for the students.

An example of the types of problems that were used by the cooperative groups is included in Appendix A.

The following are the expectations and the responsibilities that were presented to the students in the treatment group. These expectations were reviewed periodically throughout the semester.

- Students are expected to become actively involved with the mathematics that is being taught.
- Students are to take responsibility for their own learning of mathematics.
- The students are expected to cooperate with their group members, achieve a group solution for each problem, make sure that everyone understands the solution before moving on to the next problem, listen carefully to others and try to build on their ideas, share the leadership of the group, and make sure that every group member participates, and no one dominates.

The importance that each group proceeds at a pace that is comfortable for their own group members was also stressed [6]. Initially, the students were placed in groups by quiz scores; however, if the composition of a group was not conducive to a productive learning environment, the group was reorganized until a good match was found. This was accomplished by observing the groups while they worked on cooperative tasks, taking notes, and allowing the group members to evaluate themselves as well as the other members in their group as to their effectiveness in using cooperative learning techniques. If the groups were not working effectively, discussions were held and suggestions were made. If the group was still not working effectively, the instructors rearranged some of the groups.

Implementation

The Learning Together model of cooperative learning developed by Johnson and Johnson [9] at the University of Minnesota was adopted for this study. Students using this method work together in four- or five-member heterogeneous groups on assignments. The students are required to produce a group product.

INSTRUMENTATION

Testing Materials

The algebra placement test (ALGPL) is a 29-item paper and pencil test administered to all incoming freshmen to determine the appropriate mathematics course placement. The mathematics department at the university created the ALGPL to be used for the mathematics placement of freshmen.

The students in the MA 101 treatment group were given a different final exam, than their counterparts, however, all exams were different versions of computer-generated exams. The MA 101 final exam given to the MA 101 treatment group was a 50-item cumulative test created by the researcher covering all of the material taught in this course. The MA 101 final exam given to the MA 101 control group was a 50-item cumulative test created by the coordinator of the course covering all of the material taught in this course. Both exams covered the same body of material. All of the students who participated in the study took one of the eight versions of each of the tests and one of two versions of the final exam. All students involved in the study took five tests and a final exam for MA 101 and MA 111. All students in MA 111 took one of the eight versions of each of the tests and a common final exam. The MA 111 final exam was a 50-item cumulative test created by the coordinator of the course. All instructors used the same weightings for tests and the final exam when assigning final grades. The exact same standards were used each year in assigning grades. For MA 101 and MA 111, each test counted as 15% of the final grade, and the final exam was 25% of the final grade.

At the end of the 1990 Spring semester, the students in the treatment group (only) were given questionnaires to complete which examined their feelings about the use of cooperative learning in their MA 101 course and its effect on both their mathematics learning and their attitude about mathematics.

STATISTICAL ANALYSIS

A non-equivalent control-group design was chosen for this study since all of the students in a particular class had to receive the treatment, and random assignment was not possible.

Analysis of variance (ANOVA) and multianalysis of variance (MANOVA) were used to test for differences between means and interactions for HSGPA, SAT-M, SAT-V, and the MA 101 and MA 111 grades for the control and treatment groups.

The final exam for both MA 101 and MA 111 for the treatment and the control groups accounted for 30% of the student's final grade. The final grade, instead of the score on the final exam, was chosen as the measure of achievement because students in these courses tend not to perform as well as expected on final exams, and the researcher chose not to base achievement on the bases of one exam score.

RESULTS AND CONCLUSIONS

The ANOVA found no significant difference between the means for HSGPA, SAT-M, nor SAT-V for students in the control and the treatment group, therefore, the students in both groups were assumed to be the same type of student.

	CONTROL GROUP ($n = 212$)		TREATMENT GROUP ($n = 178$)	
	Mean	SD	Mean	SD
HSGPA	3.133	0.320	3.091	0.336
SAT-M	471.388	58.347	468.029	58.710
SAT-V	451.483	64.631	438.540	67.285
MA 101 Grade	4.062	0.920	3.613	1.016
MA 111 Grade	2.282	1.119	3.102	1.308

Table 1. Means and Standard Deviations for Each of the Variables.

An ANOVA showed a *significant* difference between performance in MA 111 from 1989 and 1990. The grade in MA 101 was found to be a significant factor in performance in MA 111. There was also a *significant* difference between the MA 101 and the MA 111 grades for the two years. The MA 101 grades for the students in the control group were *significantly* higher than those of the treatment group. Conversely, the MA 111 grades for the treatment group were significantly higher than those of the control group.

Source of Variation	Degrees of Freedom	Mean Squares	F Squares	Probability
Model	2	52.769	40.86	0.0001
Error	343	1.292		
Corrected Total	345			

Source	Degrees of Freedom	Type I SS	Mean Squares	F	Probability
Year	1	55.630	55.630	43.07	* 0.0001
HSGPA	1	49.908	49.908	38.64	0.0001

Source	Degrees of Freedom	Type III SS	Mean Squares	F	Probability
Year	1	62.338	62.338	48.27	*0.0001
HSGPA	1	49.908	49.908	38.64	0.0001

*Significant at $p < .05$.

An ANOVA showed a *significant* difference between the performance between the treatment and the control groups in MA 111. The MA 101 final grade was found to be a significant factor in performance in MA 111. There was also a *significant* difference between the MA 101 and MA 111 grades for the two groups.

Table 2. ANOVA Using Math 111 Grades as the Dependent Variable for Both Groups.

A scatter diagram of the MA 101 grades versus the MA 111 grades for students in the treatment group showed a positive relationship between these grades. This would indicate that the MA 101 grade a student receives has a relationship with the MA 111, and may be used as a predictor of success for MA 111 for the treatment group.

Table 4 further explains the the relationship between the grades of students in MA 101 and MA 111 for the control and the treatment groups.

Students from the control and the treatment groups who did not successfully complete MA 101, but registered for MA 111 were not eliminated from the study. The grades given to the students in the treatment group may have been more representative of the students' ability (the final exam was not multiple-choice, whereas the final exam for the control group was

MA 111 GRADES	F	D	C	B	A
A					3
B			1	6	23
C		3	6	21	34
D		1	10	19	13
F	1	6	29	26	10
	MA101 GRADES				

*Note: Each number denotes the number of occurrences at that point.

Table 3 (a). Scatter Plot of the MA 101 Versus MA 111 Final Grades – Control Group.

MA 111 GRADES	F	D	C	B	A
A			2	6	20
B		1	6	25	11
C		3	20	16	6
D	1	7	9	6	
F	2	15	8	8	2
	MA101 GRADES				

*Note: Each number denotes the number of occurrences at that point.

Table 3 (b). Scatter Plot of the MA 101 Versus MA 111 Final Grades – Treatment Group.

CONTROL

Rise in Grade

	Number	Percent
1 letter grade	4	1.89
2 letter grades	0	0.00
TOTAL	4	1.89

TREATMENT

Rise in Grade

	Number	Percent
1 letter grade	17	9.55
2 letter grades	2	1.12
TOTAL	19	10.67

CONTROL

Maintained Grade

Grade	Number	Percent
A	3	1.42
B	6	2.83
C	6	2.83
D	1	0.47
TOTAL	16	7.55

*Note 1 student maintained an an F in MA 111

TREATMENT

Maintained Grade

Grade	Number	Percent
A	20	11.24
B	25	14.05
C	20	11.24
D	7	3.93
TOTAL	72	40.46

*Note 6 students maintained an F in MA 111

CONTROL

Dropped 1 Grade

Grade	Number	Percent
A to B	23	10.85
B to C	21	9.91
C to D	10	4.72
D to F	6	2.83
TOTAL	60	28.31

Dropped 2 Grades

Grade	Number	Percent
A to C	34	16.04
B to D	19	8.96
C to F	29	13.68
TOTAL	82	38.68

Dropped 3 Grades

Grade	Number	Percent
A to D	13	6.13
B to F	26	12.26
TOTAL	39	18.39

Dropped 4 Grades

Grade	Number	Percent
A to F	10	4.72

TREATMENT

Dropped 1 Grade Grade

Grade	Number	Percent
A to B	11	6.18
B to C	16	8.99
C to D	9	5.06
D to F	15	8.43
TOTAL	51	28.66

Dropped 2 Grades

Grade	Number	Percent
A to C	6	3.37
B to D	6	3.37
C to F	8	4.50
TOTAL	20	11.24

Dropped 3 Grades

Grade	Number	Percent
A to D	0	0.00
B to F	8	4.50
TOTAL	8	4.50

Dropped 4 Grades

Grade	Number	Percent
A to F	2	1.12

Table 4. Comparison of Change in Grades By Year.

multiple-choice). The students in the treatment group appeared to be better prepared for MA 111 than their counterparts, as witnessed by their success rate in MA 111. This statement can be supported by the fact that of the 95% of the students in the control group who successfully completed MA 101 (received a grade of C or better), 46% of these students registered for and successfully completed MA 111. Of the 85% of the students in the treatment group who successfully completed MA 101, 70% registered for and successfully completed MA 111. In addition, 55% of the students in the treatment group maintained or improved their grade from MA 101 to MA 111, whereas only 10% of the students in the control group maintained or improved their grade.

DISCUSSION

The findings of this study have several implications for education and further research. First, Slavin [30] reported a paucity of studies examining the effects of cooperative learning at the college level. He also reported that the results are not as consistent as those found in studies at the elementary and junior high/middle school level. This inconsistency may be a function of the limited amount of available data and the variety of methods and subject matter used. Also, recent studies examining cooperative learning methods exclude one or both of the two elements Slavin suggests are essential in order for the cooperative learning experience to be effective: group goals and individual accountability. Slavin's comments underscore the need for further research at this level to deal with the consistency issue.

Another question raised by Slavin [30] concerns the appropriateness of cooperative learning for higher-order conceptual learning. The problem-solving focus of this study offers evidence that cooperative learning may very well be appropriate. Problem solving and cooperative learning experiences provide opportunities to extend mathematical thinking beyond that typically required in traditional instructional methods. The aim of this study, however, was not to measure the effects of cooperative learning on problem solving or on higher-order conceptual learning. The dependent variable here was success in the next mathematics course. One explanation of the results might be that because students in the treatment group worked cooperatively on problems that required higher-order thinking skills, therefore helping these students develop higher-order thinking skills. The success of the students in the treatment in MA 111 may have been due to the extension of their mathematical thinking and their ability to transfer their learning to new and different situations. Unfortunately, this inference fails to take into account the effects of cooperative learning on the learning experience of the treatment group.

Although it is possible to conclude from the data gathered that the cooperative learning/problem-solving method used with the treatment group is more effective in promoting success in MA 111, there are questions which need to be addressed in further research:

1. Is cooperative learning the primary cause for improvement in grades?
2. Is problem solving the primary cause? or
3. Is the combination of both elements necessary for consistency?

This study does not make any attempt to differentiate between the two.

Another recommendation for further research is to investigate how to use other tests or a test score, such as critical thinking or logical reasoning, to measure the effect of of the use of problem solving on developmental cognitive processes.

It may also be beneficial to examine why cooperative learning or problem-solving is effective. Does cooperative learning/problem-solving lead to consistent grades in subsequent mathematics courses because of better understanding of content, increased confidence in mathematics skills (see goals of NCTM), enhanced math attitudes, or is some other factor involved? Each of the factors above is important and each is a reported advantage of using cooperative learning and/or problem-solving techniques.

The study itself can be extended to follow students to subsequent mathematics classes to see if differences between treatment and control groups persist and for how long.

In its *Curriculum and Evaluation Standards for School Mathematics* [18], the National Council of Teachers of Mathematics articulates five general goals for all students. Although the Standards were developed for grades K-12, these goals are equally applicable to students enrolled in mathematics classes at any level. NCTM proposed that students:

1. learn to value mathematics,
2. become confident in their ability to do mathematics,
3. become mathematical problem solvers,
4. learn to communicate mathematically, and
5. learn to reason mathematically.

The cooperative learning/problem-solving approach used in this study provides a framework to address each of the above goals. Students communicate mathematically as they work cooperatively in small groups to solve problems. True problem-solving as envisioned above requires that students learn to reason mathematically. Through this process it is hoped that students learn to value mathematics and become confident in their ability to do it. The grade consistency of the treatment group and the comments from these students about the cooperative problem-solving approach seem to bear this out.

Although there has been much research in the area of cooperative learning, according to Davidson and Kroll [7] there are a variety of important research questions related to cooperative learning in mathematics that still need to be addressed in further research. One such question is whether cooperative learning is more effective instructional strategy than more traditional methods (competition and individualistic goal structure) of instruction. In addition, interest has arisen concerning the conditions under which cooperative learning improves student achievement. Further research is needed to determine which conditions and which of the various combinations of group rewards and individual accountability are necessary for improving student achievement. Lastly, another area of concern involves the benefits (either cognitive or affective) resulting from engaging students in cooperative learning. These benefits include such variables as student behaviors, teacher behaviors, heuristics used in problem solving, and group dynamics.

APPENDIX A

Example problems used by the treatment group during cooperative learning classes.

Number Systems

1. A grasshopper jumps along a number line as follows: He starts at zero, moves forward one unit, then backward two units, forward three units, backward four units, forward five units, and so on. Where will he be after

a) 15 moves? d) n moves?
b) 2,000 moves? e) on what move will he hit the point +100?
c) 2,001 moves? f) will he eventually hit every integer?

2. Insert the operation signs (+, −, ×, ÷) and parentheses in order to make the following statements true: Ex: 3 1 2 = 1

$$(3-1)-2=1$$

a)	12	4	2	2			=	4	d)	8	6	4	2	=	4
b)	4	10	2	3			=	36	e)	8	6	4	2	=	7
c)	2	7	2	3	8	10	=	−3	f)	8	6	4	2	=	1

Solving Equations

1. A large science class is held in a lecture hall with theatre seating for a capacity of 180. If four times as many seats are occupied as vacant at a recent meeting, how many students were present?

a) Men outnumbered women by seventy for another class in the hall. How many women attended if the hall was full?

b) Two classes shared the hall for a special lecture. One class is twice as large as the other. How many students are in each class if together they occupy only 75% of capacity?

c) The theatre seating is sold out for the screening of a film for campus and community. There are a dozen less than twice as many students as others attending. How many students see the film?

2. My car pool begins at my house I pick up (in order) John, Terry, and Amber before reaching work, a distance of 39 miles. It is 1 mile farther from my house to John's house than it is from John's to Terry's and 1 mile farther from Terry's house to Amber's house than from Amber's house to work. How far is it from my house to John's house if Amber lives the same distance from work as I live from John's house?

Linear Equations

1. Suppose the scoring in a football game is simplified to 7 points for a touch down, 3 points for a field goal, and no other scoring.

a) Write and equation that represents the total score.

b) What combinations of touch downs and field goals would give a score of 46? of 42?

c) What scores are impossible to achieve?

2. Find the function $f(x)$ that is represented by each sequence of calculator keys (using algebraic logic). Enter a number X, then press the keys in sequence.

a) [×] 4 [+] 3 [=]

b) [x^2] [×] 2 [+] 3 [−]

c) [x] 2 [+] 1 [=] [√]

d) [Y^x] 3 [×] 2 [+] 3 [=]

3. Mr. Smith lives 15 miles from work where he had free parking. Some days he must drive to work so that he can call on clients, but other days, he can take the bus. It costs him 20 per mile to drive the car and 80 round trip to take the bus. Assuming there are 22 working days in a month, how many days can he drive and still hold his transportation cost to $30.00 a month?

4. A typical scoring system on a multiple-choice test where each item has five alternative answers is: (number correct) - 1/4 (number incorrect). Your score on a 50-question test is 35, and you answer every question. How many did you get correct?

Linear Inequalities

1. In the city, teachers are paid $14,000 to start and $500 for each year of teaching experience. In the suburbs, teachers are paid $12,500 to start and $700 for each year of teaching experience. How many years of teaching experience must a teacher have in order to make the salary in the suburbs higher?

2. Your club wants to hire a band for its spring dance. Band A will play for $300 plus 40% of the revenue. Band B will play for $600. If you expect 500 people to attend, what is the most you could charge for each person if you want to make Band A the better buy?

3. The sum of four consecutive integers is between 65 and 80. What are the possibilities?

Rational Expressions

1.

a) If a number is doubled, what happens to its square root?

b) If a number is tripled, what happens to its square root?

c) If a number is increased 100 times, what happens to its square root?

2. Use your calculator to find the sequence :

$$\sqrt{2},\quad \sqrt{\sqrt{2}},\quad \sqrt{\sqrt{\sqrt{2}}},\quad \sqrt{\sqrt{\sqrt{\sqrt{2}}}},\ldots$$

a) continue for at least 25 times

b) what happens? How can this be true?

c) write the above sequence using rational exponents

d) what is the general form for the nth team of this sequence of numbers?

3. Repeat exercise #12 for cube roots.

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