

From Boxes to Bees: Active Learning in Freshmen Calculus

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Abstract—The vehicles of education have seen significant broadening with the proliferation of new technologies such as social media, microblogs, online references, multimedia, and interactive teaching tools. This paper summarizes research on the effect of using active learning methods to facilitate student learning and describes our experiences implementing group activities for a calculus course for first year university students at Singapore University of Technology and Design, a new design-centric university established in collaboration with Massachusetts Institute of Technology (MIT). We describe the educational impact of different pedagogical techniques, such as real-time response tools, hands on activities, mathematical modeling, visualization activities and motivational competitions on students with differing learning preferences in a unique cohort classroom setting. Based on faculty reflection and survey data, we provide guidelines on how to adopt the right set of active and group learning techniques to handle the changing learning preferences in the current and future generation of students.

Index Terms—active learning, group learning, calculus, pedagogy, real-time response

I. INTRODUCTION

Active learning is an instructional method that allows engagement in the learning process and requires students to think about and be actively involved in meaningful learning activities [10]. Although active learning can include conventional activities such as assignments, more effective approaches involve focusing on student activities and engagement in the learning process, instead of traditional lectures where students passively receive information [6]. Studies have shown that students can also master difficult subjects more effectively using active learning activities [3], [5], [9].

The objectives of this paper are to summarize the effects of active approaches to student learning and describe our experiences in implementing new active learning strategies to teach calculus for first year university students at Singapore University of Technology and Design (SUTD), a newly established university in collaboration with MIT. We evaluate different pedagogic approaches including active learning, interactive learning with real-time response, modeling activities, and group activities for teaching calculus and provide recommendations on how to tailor these activities for students with different learning preferences.

The paper is organized as follows. Section II provides the background of the course and the university. Section

III describes a typical active learning approach used in the cohort classroom. Sections IV and V provides details on the types of activities that were implemented. Section VI presents the results before concluding in Section VII.

II. BACKGROUND

As SUTD is a unique design-centric university, the current best practices in educational theory and design are being used in conjunction with interactive tools and hands-on activities. Design as a discipline cuts across the curriculum and provides a framework for the research and educational programs. The curriculum is taught through a unique pedagogy. This paper focusses on the application of this pedagogy to the Advanced Math I (Calculus) course conducted as part of the first year core curriculum¹. The authors of this work are the faculty members who have devised and supervised the activities in the first run of the course in Spring 2012.

Advanced Math I is an introductory course on single-variable calculus, which includes the derivatives and integrals of real-valued, single-variable functions and their applications. The entire course is divided into three units as follows:

Unit 1 Derivatives: Functions and Derivatives; The Exponential Function; Differential Equations of Growth; The Derivative of the Sine and Cosine; The Product, Quotient, and Chain Rules.

Unit 2 Applications of Derivatives: Inverse Functions and Their Derivatives; Implicit Differentiation; Maximum, Minimum, and Second Derivative Test; Limits, Continuity, l'Hôpital's Rule; The Mean Value Theorem; Linear and Quadratic Approximations and Taylor Series; Newton's Method.

Unit 3 Integrals and Their Applications: Riemann Sums and Definite Integral; Fundamental Theorem of Calculus; Applications: Areas, Volume, Arc Length, Surface Area; Properties of the Definite Integral and Average Value; Numerical Integration; Probability and Calculus; Techniques of Integration.

While the content of Advanced Math I is similar to Freshmen Single-Variable Calculus courses offered in most engineering schools, this course is taught through a unique

¹www.sutd.edu.sg/curriculum_subject_descriptions.aspx

pedagogy at SUTD, referred to as *cohort-based active learning*. The students are organized around small learning communities, called cohorts. The course material is delivered through a set of activities which are specifically designed to take advantage of the small-sized cohorts by facilitating useful interactions between faculty and students that belong to the same cohort. Due to this interaction and with careful use of real-time response systems, faculty members are able to collect continuous feedback from the students and make adjustments to improve the learning experience as the course proceeds. One of the aims of this paper is to identify the activities that are well-received by the students in this course together with the reflections of the faculty members who have prepared and also delivered them.

The course was structured as a one hour lecture followed by four hours of cohort classes (two sessions of two hours each) per week. The lectures were delivered by a senior faculty member, while each cohort class was run by three junior faculty members. The course structure was designed so as to promote active learning. This was achieved by designing the cohort classes as a series of carefully selected activities and making use of recent advances in education technology. The course was run over 14 weeks (with a 1 week break) with a total enrollment of 323 students. The students were divided into 7 groups of about 45-48 students for each cohort class. While technology enabled active learning approaches have been previously implemented (see Freshmen Physics classes at the MIT Technology Enabled Active Learning (TEAL) classroom [1] for an example), to the best of our knowledge it has not been implemented for Freshmen Mathematics classes. Our experiment provides the findings implemented through an active learning approach for Math at SUTD.

III. ACTIVE LEARNING APPROACH

One of the objectives of mathematics education at the undergraduate level, especially in the freshmen year, is to convert the students into free and creative thinkers. Unfortunately, in many countries where the university admission system is based heavily on generalized exam scores, high school students tend to memorize possible question types and their solutions instead of learning the basic and most important concepts. It is not an straightforward task, even for seasoned educators, to change the mind set and studying habits of these students. We observe that cohort-based active learning when planned correctly can make a difference in increasing students' thinking abilities. The key element in conducting successful cohort classes is to devise the activities carefully and understand how it fits different student preferences. We typically dedicate each cohort class to a main concept and perform activities that can often be divided into five groups as shown in Figure 1. In some cases a full length cohort activity spans across the shorter cohort activities.

The details of the cohort class are provided next:

Mini-lecture: Mini-lectures are used in our cohort classes in order to recall results discussed in the main

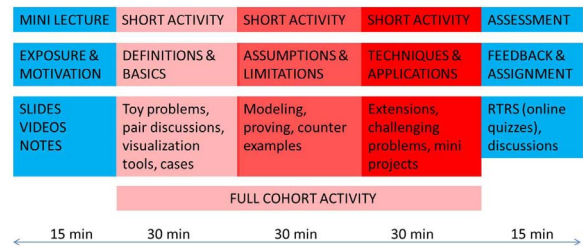


Figure 1. A typical 2 hour cohort class for Advanced Math I

lecture as well as new results that will be used in the current cohort. These lectures are given by one of the faculty members and may include notes on the whiteboard, slides, videos, demonstrations using physical objects, or online tools such as applets. It is critical that these lectures are short, concise, and well-timed. We make use of them at the beginning of the cohort classes or between activities depending on the material being covered.

Short Activity (Definitions and Basics): The short activities are usually of duration up to 30 minutes. The goal here is to provide exposure and motivation to the main concepts that will be discussed in the cohort class in an intuitive, interactive, and engaging way. They may include group discussions, graphing exercises, and hands-on activities. These are often shorter in duration than the latter activities.

Short Activity (Assumptions and Limitations): These are the core of a cohort class where the concept is treated more rigorously. They are typically more challenging and sometimes performed over a longer time period. Typical activities are modeling new problems, proving main results, and understanding the limitations of the concepts. These type of activities are ideally followed by a concise but rigorous treatment of the subject by a faculty member on the board.

Short Activity (Techniques and Applications): These are performed at the end of the cohort classes and usually devoted to either extending the results or using them in a specific application. Sometimes, challenging exercises can be given at this stage to enhance and deepen the learning as well as increasing the curiosity and excitement. In some cases the shorter activities are combined to a full length cohort activity where the distinction between the different phases is not as obvious.

Assessment: Throughout a cohort class various assessment tools are used to collect feedback about the effectiveness of the teaching techniques. We make use of real-time response systems in order to collect quantitative and immediate feedback. Nevertheless individual and group discussions are also very valuable for collecting qualitative feedback. The abundance of personal interactions is an inherent property of cohort-based active learning and one of its main advantages.

In the next two sections, we describe some of the cohort type activities that were implemented together with details of the real-time response systems.

IV. SHORT COHORT ACTIVITIES

A. Calculating Simple Derivatives using Lego

A visualization activity was used with Lego construction bricks, as shown in Figure 2, to provide a geometric interpretation of a basic calculus result - The derivative of x^3 is $3x^2$. The Lego based construction is inspired from an example in [7] and validates physically why this result is true. Lego pieces are used to build a cube of size $(x + \Delta x)^3$ enclosing another cube of size x^3 . The difference between the volumes of the larger and smaller cubes can be decomposed into seven pieces: a cube of size $(\Delta x)^3$, three rectangular cylinders of size $x(\Delta x)^2$, and 3 “squares” of size $x^2(\Delta x)$. As Δx goes to zero, the difference between the volumes of the two cubes (in the limit) is three squares each of size x^2 (three faces of the $(x + \Delta x)^3$ cube). Students were receptive to such in-class demonstrations as many prefer visualization examples. Such an explanation, though not representative of all possible situations, proved very effective in helping students grasp the abstract mathematical concept in a physical and intuitive way. Although we have performed this activity during the lecture this year, we believe that this would be a good cohort activity as well when students use the Lego pieces themselves.



Figure 2. Calculus lecture using Lego demonstration

B. Relationship between a Function and Its Derivatives using Graphing Tools

Helping students grasp the abstract mathematical concepts in a more physical or intuitive way can dramatically boost their interest and confidence in the subject. In explaining the derivative of a function, students were engaged in a series of “graphing” exercises in which given a particular graph on a sheet, paired students drew a derivative graph purely based on the shape of the original graph. Such activities have been advocated as part of the “Reform Calculus” initiative [2]. Around 15 graphs were given to the students and were either based on real functions or simply drawn to make

the drawing of its derivative graph more interesting and challenging (see Figure 3 for a sample graphing sheet as drawn by a student). After the paired exercises, one student from each pair drew it on the whiteboard while getting feedback from the class. Rather than thinking of a derivative as a mathematical formula, the students started thinking of it as a tangible graph just like the original graph but representing the rate of change of the original graph. In subsequent classes, simple exercises for drawing the original graph based on the derivative graph was also undertaken. Many students found this very challenging but at the same time showed focus and interest. We have observed throughout the semester that the strong graphical intuition obtained early in the course became an important factor for later stages. When given a problem, students immediately started thinking of a function’s shape and used this intuition to understand the problem more clearly.

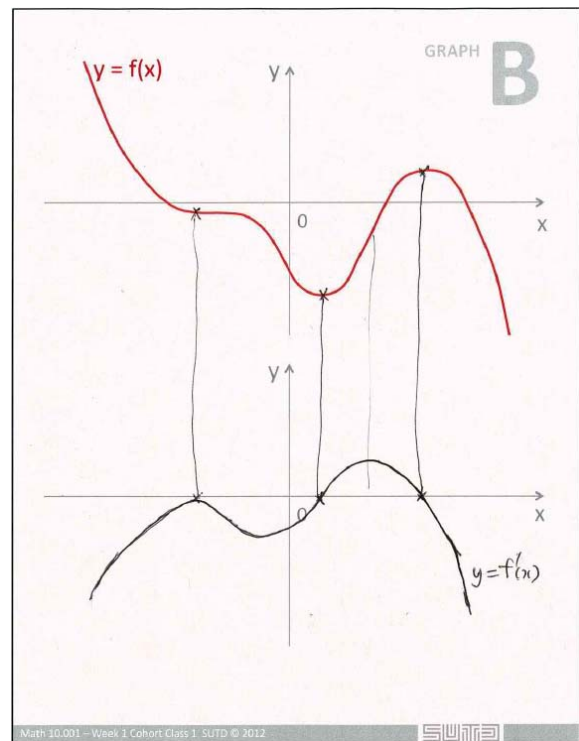


Figure 3. A sample graphing sheet: given a graph of a function (red line), a student drew a graph of the derivative function below only by looking at the shape of the original graph.

C. Volume and Surface Computation Using Method of Discs and Shells

This activity was introduced in the component of the course which dealt with the determination of volumes of solids of revolution. In this activity, the students were tasked to determine the volume of a sphere through dissecting the sphere into both discs and shells. It allowed the students to apply their knowledge of Riemann Sums through approximations of the volumes of the sphere. After the hands-on activity, the students were required to derive the analytical

formula of the sphere through the two methods and hence discover that both approaches are equivalent.

The goals of this activity were to have students understand the equivalency of determining volumes using the methods of discs and spheres, have hands-on activities to visualize both methods, generate a better intuition on how to apply both methods, allow students to understand better through different modes of learning, and understand the concept of Riemann sums for the volume of a sphere and assumptions in over- and underestimation of volume.

Figure 4 illustrates how the activity was conducted with modeling clay. Students were required to build an approximate sphere using either different sized cylinders or discs. This allowed students to visualize the two methods and gain better understanding about the underlying concepts of the methods. The students would then proceed to determine the general formulas for solids of revolution from the two methods and from them get back the exact form of the formula for a sphere.



Figure 4. Activity using modeling clay to visualize volume using discs and shells

V. FULL COHORT ACTIVITIES

A. Modeling Population Growth with Simple Differential Equations

The students were given a task to work on mathematical modeling of a real world phenomenon. In particular, with guidance from instructors, the students were asked to develop a model for population growth using a basic differential equation. The objectives of this task were two fold. First, students would acquire the mathematical skills to work with simple differential equations of the form $\frac{dy}{dx} = cx + s$: an important topic in calculus. Second, the students would learn and experience the general steps in modeling a real world phenomenon with mathematical concepts and techniques, and understand how to use the model to study the effects of different components and to make predictions about future.

At the start of the class, the students were asked to brainstorm factors that would affect population. Some factors that were identified are food, space, climate change, etc. Students were also asked to discuss the particular characteristics of population growth, namely, (i) whether the rate of change of population would depend on the current population size, and (ii) whether there would be any upper limit on the population size that our environment can support. Based on the discussion, students were asked to sketch a plot of population versus time, which would be compared with the mathematical models developed during the task.

During the two-hour class, students were asked to apply differential equation techniques to model the population. U.S. population data available from the Internet were used for comparison. First, students were asked to plot the U.S. data using Excel, discuss the characteristics, and make comparison with their own population sketch. Simple first-order linear differential equations were then reviewed, based on students' discussion on how the rate of change of population depending on the current population size. Students were asked to solve this simple differential equation, estimate the model parameter, and compare with the U.S. population data. Students found that while this simple model could account for the initial population growth, there was considerable discrepancy as time continued. Students were then asked to identify the deficiency of this model and methods to improve it.

Students spent the full 2 hours step-by-step going through increasingly complex levels of differential equations, starting with a simplistic linear population growth equation to estimate the future world population with spreadsheets, then being introduced with "competition terms" to tame the exponential growth again with spreadsheets, then using a further revised differential equation to more accurately predict future population. Based on this they could point out that this simple linear differential equation model did not consider resource competition in the population and therefore would lead to over-estimation of population size. Students were then asked to introduce a quadratic competition term into the model, leading to a non-linear differential equation. Students were then asked to solve the equation and plot the model against the U.S. population data, as shown in Figure 5. Students found that with the introduction of an additional component the model was much improved and matched well with the actual population data. Several characteristics of the model were then discussed. Students were also asked to use the model to predict the future population. This gradual exposure to increasingly complex modeling within a 2-hour session was planned to show how mathematical models are being used in practice while hoping to engage students through the prediction activity.

B. Understanding the Mean Value Theorem with a Step by Step Approach

The Mean Value Theorem (MVT) states that given two points on the real line and a function which satisfies certain assumptions on continuity and differentiability, there exists

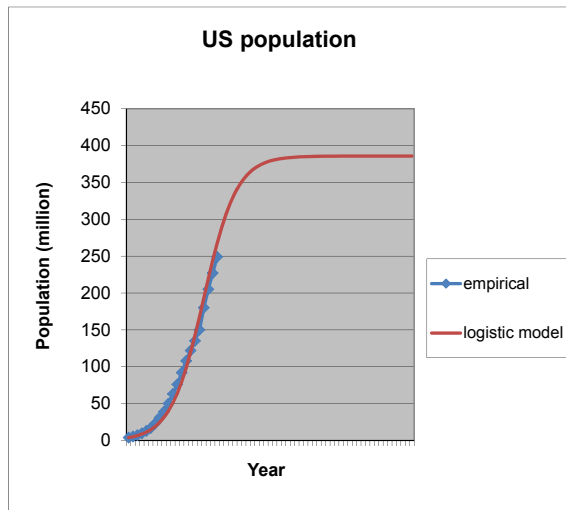


Figure 5. Mathematical Modeling Task: Students are guided to develop a non-linear differential equation (logistic model) to model the U.S. population

a point in the open interval between these points for which the derivative of the function evaluated at this point is equal to the average (mean) function value of the two end points. In a typical classroom, this is taught by stating the theorem, sometimes providing a proof, and maybe a corollary of the theorem in a short amount of time, where students passively follow the instructor and take the result as granted without usually paying much attention to the conditions of the theorem or the richness of this statement. In our classes, we used a more active approach where students were exposed to the theorem first in an intuitive manner (Short cohort activity), then rigorously by focusing on the assumptions of the theorem (Short cohort activity), and finally by discovering corollaries of the theorem (Short cohort activity). The first activity was designed to understand the result intuitively by applying it to motion. The students were told that “If a person travels from home to work which are twenty kilometers apart in twenty minutes and the speed limit in the city is 40 km per hour, then the driver is under the risk of getting a speeding ticket during this journey” and assigned to the task of discovering why this is a fact (see Figure 6). They have discovered that average speed during that time was 60 km per hour. To get at that average speed, the car either has to go at a constant 60 km per hour during that whole time, or, if it goes slower at one moment, it has to go faster at another moment as well (and vice versa). Therefore, at some time during this journey, the car must have been traveling at exactly 60 kms per hour; that is, it was traveling at its average speed which is greater than the speed limit. In other words, students investigated the difference between instantaneous speed and average speed. Then faculty members facilitated a short group discussion and formally introduced the MVT after the discussion with a mini-lecture. In the second activity, students revisited the previous scenario and discovered that their conclusions were

in line with the result of the MVT. Afterwards, students were assigned the task of identifying the assumptions of the theorem and show why each of these assumptions is critical. Supervised by the faculty, each group of students constructed simple counterexamples where the assumptions and the result of the theorem fail. The MVT can be applied to a function if the function is differentiable over the open interval between the two points and continuous over the closed interval between the same points. Hence in particular, students spent time finding out simple discontinuous or non-differentiable functions that violate the conditions and also the result of the theorem. Faculty members used this opportunity to collect feedback about how well these two fundamental topics were understood by the students and corrected any misconceptions if necessary. They also emphasized that there can be cases where the result holds but the conditions fail. This activity was designed to deepen thinking abilities of the students and expose them to the critical and constructive perspective one should take when approaching a new result. This was further emphasized by guiding the students in proving a new result using the MVT theorem, in particular, proving that if the derivative of a function is positive over an open interval then the function must be strictly increasing over the same interval. We believe that when cohort classes are carefully planned as above to combine various types of activities and enhanced with discussions, cohort-based active teaching can be very effective not in only more applied subjects, but also in fundamental classes such as first year calculus even when a rigorous theorem is being discussed.

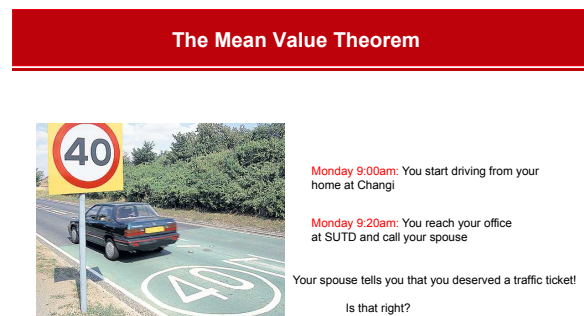


Figure 6. An Intuitive Understanding of MVT

C. Modeling and Optimization using a Hands-on Box Building Exercise

In this activity, students were asked to build an enclosed box from a rectangular cardboard with maximum volume. This activity was inspired from the article on ‘Thinking outside the box - or maybe just about the box’ in [4]. A group of five to six students were given cardboard, and tasked to build their box within an hour. The students were told only a maximum of 6 cuts are allowed, and were given scotch tape for them to stick different pieces of cardboard together. Students were not told beforehand that this is a two-variable problem. By exploring the box design, students

needed to think through additional constraints to translate the problem into a single-variable problem. The problem was designed to be open ended so that students themselves had to discuss and identify and ask back questions to the instructors, in order to train the students for tackling often ill-defined real-world problem situations.

By the end of one hour, we presented a standard solution that some students had found and demonstrated how to solve such a problem analytically. For the cardboard, of width 64 cm (i.e. A in Figure 7) \times height 51 cm (i.e. B in Figure 7), we include an additional variable of the depth of cut L , and determine that the position of the first cut should be $2L$ away from the side to maximize the use of cardboard. This additional constraint is selected because we assume to perform 6 cuts as indicated in Figure 7. By doing so, we translate the problem into a single variable L , and we can solve for the optimal L value that maximizes the volume of the box. The optimal value of L is 6.4 cm, with volume of 9388 cm^3 .

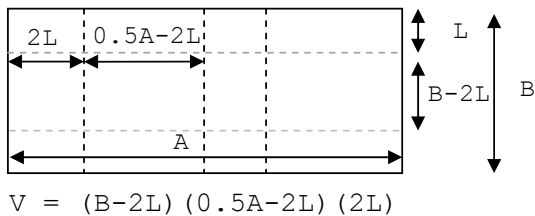


Figure 7. Box construction activity - Optimal solution

Nonetheless, an important finding from this activity was that students in fact did think out of the box, as the title of the article suggests and present innovative solutions. For example a group found that it is possible to have a box with larger volume than the standard solution presented above with fewer cuts. The solution is shown in Figure 8 where by choosing $C = 19.25 \text{ cm}$ and $D = 25.5 \text{ cm}$, the volume of the box is 12517 cm^3 which is 33% higher than the standard solution.

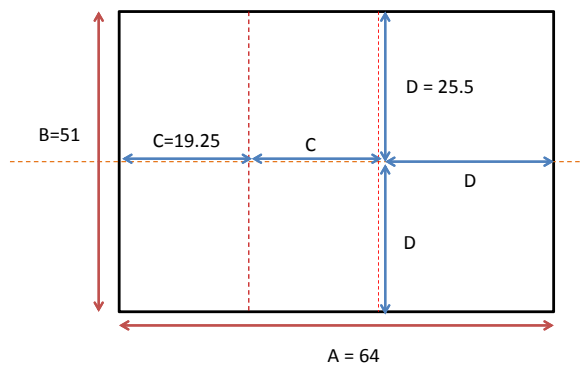


Figure 8. Box construction activity - Student found solution

From this activity, it was clear that with more open ended

questions, students have the freedom and will to find new solutions. Most importantly the instructors need to be able to support such learning that happens beyond the intended goals of the activity.

D. Computing Indefinite Integrals with Integration Bee

Integration Bee is a popular university competition where contestants are asked to evaluate definite and indefinite integrals. The Integration Bee contest is modeled on its more famous counterpart – the Spelling Bee contest where contestants are asked to spell words. While the origins of Integration Bee contest are not clear, MIT has implemented annual contests for at least twenty years during their Independent Activities Period. The basic philosophy of the Integration Bee contest is to get the analytically inclined students with a love for competition to face off with tough integral evaluations.

At SUTD, we launched the Integration Bee contest across the entire freshmen batch in the penultimate week of the course. The competition was held in two rounds with Round 1 consisting of 16 easy integrals (each to be evaluated in 1:30 minutes) and Round 2 consisting of 12 hard integrals (each to be evaluated in 2 minutes). There were two main differences in our implementation of the Integration Bee contest from contests held elsewhere (a) All the 323 undergraduate students participated in the Integration Bee contest. (b) The format was a group competition where each cohort class of 48 students was divided into three groups of 16 students each. In Round 1, two contestants from by each group competed across two integration questions. No help from other group members was allowed in this round. Thus all students participated in Round 1. A snapshot of Round 1 is provided in Figure 9. In Round 2, groups nominated contestants to face off on the harder questions. In this round however all students were allowed to help their teammates.

The Integration Bee contest made for a enhanced learning experience fostering both collaborative and competitive spirit among the students at the same time. Also the group format helped all students participate without the pressure of losing face in front of their friends. The competition also gave some of the stronger students to exhibit their prowess in mathematics. It was a heated, competitive and exciting activity where strong students amply showed off their math skills while weaker students reviewed the integration part of mathematics in a different and memorable way.

E. Assessment Tools (Real-Time Response System)

Teaching with a new style of pedagogy requires using novel mechanisms for formal and informal evaluation as well. In addition to the formal tools we used in the course, we also leveraged a real-time response system (RTRS), Learning Catalytics (LC), delivered over the Internet, which was previously used for the integrated learning program at SUTD [8]. Beyond a typical “clicker” system, this RTRS allowed a variety of question types including multiple choice, short answer, numeric short answer, and drawing. These alternative question styles permitted a variety of concept



Figure 9. Snapshot of Integration Bee

level questions to gauge the students' grasp of the fundamental ideas. It was possible to use the drawing function to represent curves drawn on top of an axis and grid. The RTRS also has question types that can be used to collect data from students - such as forming a word cloud from word responses or a statistical distribution based on numeric inputs. Thus we were able to crowd-source answers or use it as a brainstorming tool. The system was also used near the end of the each class to help students stretch their minds with what they have learned by asking somewhat simplified but thought-provoking questions (e.g. "will the universe keep expanding when its radius is observed expanding but the second derivative is negative?"), helping realize the potential usefulness of thinking about phenomena with the concepts and tools learned during the classes.

Implementing a RTRS into a classroom is an investment due to the time required before the class to generate questions, in class to collect the responses, and after class to analyze the data. We aim to utilize the findings in a variety of ways. Conventional clicker systems are used to assess the current understanding of students during the class session. Although we use this tool to guide our real-time teaching, we have and plan to continue to use it as feedback on our course development. One of the most compelling ways to use the RTRS data is to help determine the lessons that were decisively effective or decisively non-effective. Between these two categories is, of course, a grey area, however, from the data it is quite clear the sessions that were quite successful (nearly 100% correct) to relatively unsuccessful in delivering the main ideas (50% or less successful). Moreover, because we evaluate with several questions in each session, we can actually get higher resolved data on which concepts within the lesson need attention. As our pedagogy is built around activities that focus on particular topics, we can use the RTRS responses to determine which activities need to be adjusted the next time the course is taught. Using the RTRS is much more effective than an out of class survey because the ideas are fresh in the students heads and we also achieve a near 100% participation, which gives us more complete

data. We are also experimenting with running the questions before the activities or after the activities and also repeating concept questions one or two sessions later to determine the activities that yield long term retention.

VI. RESULTS & DISCUSSION

Pre-course and post-course surveys were conducted to better understand the students' learning preferences and the response to the cohort activities for students with different learning preferences.

A. Pre-Course

A total of 256 students (79%) responded to the course survey conducted at the beginning of the term. Figure 10 shows the results of the students' preference of individual or group learning. A majority (70%) of students surveyed selected both individual and group learning, with only 9% of students preferring group learning exclusively. This seems to indicate that most students were open to engaging in both types of learning activities - individual and group.

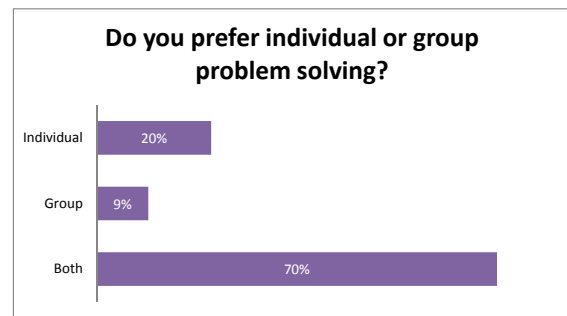


Figure 10. Individual or group learning preferences (Pre-course)

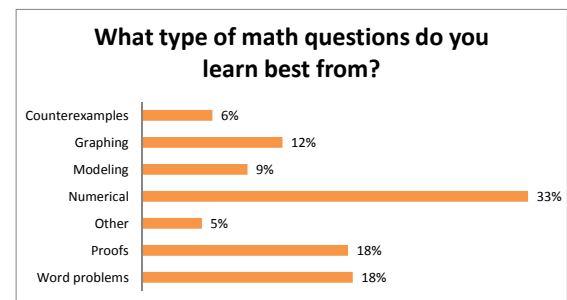


Figure 11. Preference of type of math questions (Pre-course)

Figure 11 shows the results of the students' preferences on the types of math questions they prefer. The more popular type of questions were numerical (33%), proofs (18%) and word problems (18%) and the less popular were counterexamples (6%). Examples of other types of questions include "more visual and conceptual questions", "question and answer pairs to analyze approaches, techniques, methods, thinking and apply these in future problems", "simple concept questions before moving on to guided advanced

questions, and finally, math problem sets”, “counter examples with step by step explanation”, “seeing the overall picture”, “diagrams, graphs and several problems of the same application of that particular topic”, and “all of the above”. The preferences for the types of math questions are as expected as majority of the freshmen students at SUTD have gone through a rigorous schooling curriculum that emphasizes problem solving with a heavy focus on examinations.

B. Post-Course

The post-course survey was done six weeks after the Advanced Math I course was completed. The survey was carried out in the first couple of weeks of the second term when students were taking Advanced Math II. The number of students who answered in the post-course survey was 67 (20%). The smaller number of post-course surveys taken is reflective of the fact that this survey was not compulsory. However since the survey was taken several weeks after the course was completed, it is also expected to be highly reflective of student’s true experiences.

The students’ preference for the type of math questions after the course is provided in Figure 12.

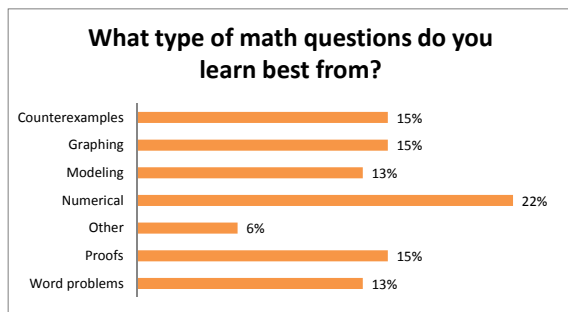


Figure 12. Preference of type of math questions (Post-course)

Based on the pre-course and post-course survey results, we observe that the percentage of students who prefer the more popular pre-course math questions reduces post-course and vice versa. For example, the percentage of students who preferred numerical questions reduced from 33% to 22%, proofs reduced from 18% to 15% and word problems reduced from 18% to 13%. On the other hand, the less popular type of math questions in the pre-course survey became more popular such as counterexamples from 6% to 15%, graphing from 12% to 15% and modeling from 9% to 13%. A possible explanation for this phenomenon is that the active learning cohort type activities provide students exposure to the importance of counterexamples, visualization and modeling in mathematics. These are generally less emphasized on in high school curriculum.

Students also selected the methods that positively affected their learning in mathematics. Students were allowed to select more than one preference. Figure 13 shows the results of the learning preferences, of which the most positive preferences were individual problem solving (67%), active learning (63%), and one-on-one tuition/ tutoring (54%).

Other preferences include “mostly visual and/or tangible explanations”, “pre-reading materials”, “practice”, “guided steps”, and “mock examples”.

To understand partly the effect of the cohort activities, Figure 14 shows the results of the preferences of cohort classes. Each student was asked to choose all cohort classes that were most useful in terms of learning. The top preferences were for Mean Value Theorem/ l’Hôpital’s Rule (55%), Applications of Areas, Volume, Arc Length, and Surface Area (55%), Functions and Derivatives(43%), Maximum, Minimum and Second Derivative (43%). These cohort classes partly map to the full cohort activity B, short activity C, short activity B and full activity C discussed in Sections IV and V respectively. Less popular was the full cohort activity A on modeling with differential equations. One possible reason is that while the modeling activity was introduced early in the course (Week 2) as a motivation for the subject of calculus, students preferred doing the basics of calculus first before dealing with more complex applications.

Students were also asked to rate some of the cohort activities directly based on how they helped in learning the concepts, on a scale of 1 to 5 (1= poor; 2= average; 3=good; 4=very good; 5=excellent). Figure 15 shows the average rating of the activity preferences, of which the most positive activities were competitions like the Integration Bee (3.82), build a box activity (3.56), and finding volume of a sphere with disk and shell methods (3.42).

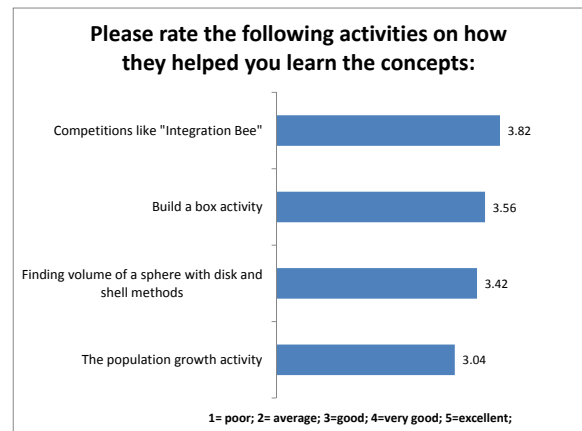


Figure 15. Summary of activity preferences (Post-course)

To understand the effect of the learning activities on students with different learning preferences, we provide in Table I the average rating of the activities for student groups that expressed preferences for different type of mathematics questions. From the table, we observe that students with more traditional learning preferences such as numerical, word and proofs prefer activities such as the Integration Bee that have well-defined answers followed by Box Building and Disc/Shell activities. However students who like modeling exercises clearly exhibit preference for more open-ended activities such as the Box Building and Disc/Shell exercise. This provides guidance on how cohort style activities can be designed to students with different learning preferences. Our

What is your preferred way(s) of learning for mathematics?

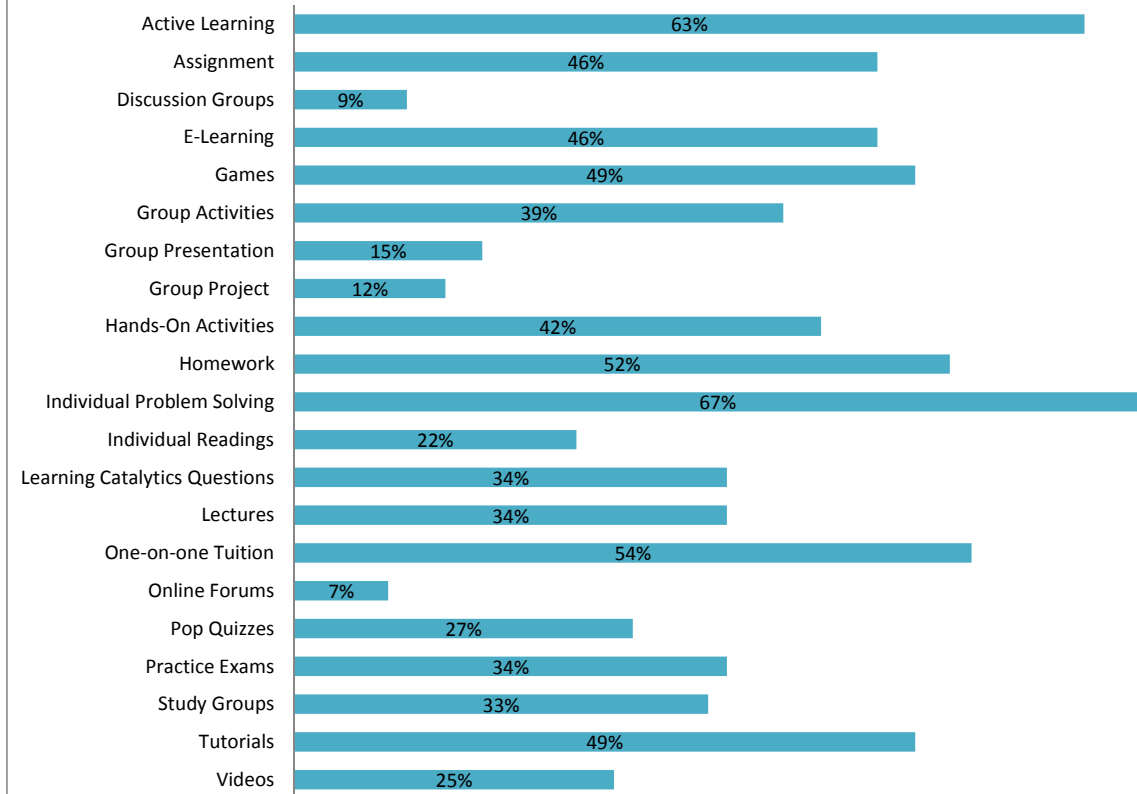


Figure 13. Summary of learning preferences

Which cohort class(es) did you learn the MOST (check all that apply)?

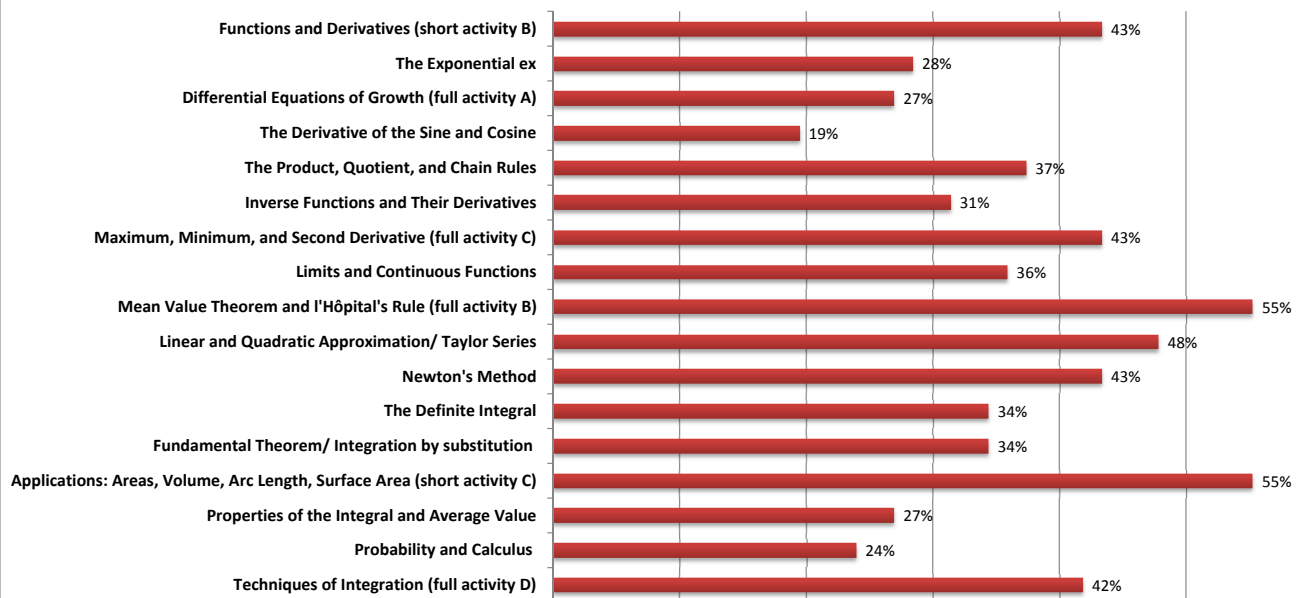


Figure 14. Summary of topic preferences (Post-course)

Table I
AVERAGE ACTIVITY PREFERENCE RATINGS FOR STUDENTS WITH
DIFFERENT LEARNING PREFERENCES

Preference	Population	Box Building	Disc/Shell	Integration Bee
Numerical	3.2	3.5	3.4	3.8
Graphing	2.8	3.5	3.5	3.9
Modeling	3.5	4.3	4.0	3.5
Word problems	2.8	3.4	3.1	3.6
Proof	3.1	3.1	3.3	4.1
Counterexamples	3.0	3.3	3.0	3.8
Other	2.2	4.1	4.1	4.0

own belief is that it is good to have a broad range of types of activities especially when students have varied learning preferences.

Eighty-four percent (84%) of students surveyed indicated that Learning Catalytics (LC) helped them in their learning. Some comments were “I think LC is good in providing some initial questions about the topic, allowing us to question some fundamentals”, “The questions are good. We can learn the little tricks that questions have”, “I hope to see the LC questions and correct answers after class for later review”, “questions are not wordy and are easy to comprehend”, “by releasing LC solutions for all the questions we see so that we can appreciate the concept questions better”. Some students feel that LC questions should not be overused “because it’s better to explain on the board by hand, especially for calculus”, and some preferred to “have more interactive teacher to student activities rather than to interact with a software”. Many students requested more time to answer the questions, and to give advance notice when LC is being used, as it takes time to turn on their devices and log into the system. A student suggested a different RTRS system which uses a remote control unit because LC requires students to bring their laptops or electronic devices. Also, a student suggested allowing LC “to serve like a forum, whereby students can pose queries during the clicker questions, allowing for more avenues to test assumptions”.

Overall, the students rated activities such as the Integration Bee and Box Building highly, as well as more technical concepts such as the Mean Value Theorem/ l’Hôpital’s Rule. The results indicates that when properly designed the active learning activities can help increase students’ motivation to learn in calculus.

VII. CONCLUSIONS

In this paper, we review the types of active and group learning techniques that were used for a first year calculus course for students in a new design-centric university established in collaboration with MIT. We used a variety of pedagogic techniques in the classrooms, such as the real-time response system Learning Catalytics, modeling activities using Excel, hands-on group activities such as box construction to learn optimization and integration with the disc

and shell methods, and competitions such as Integration Bee. We summarize the results of student’s learning preferences in the activities as well as more general modes of learning. Students were mostly responsive to the new methods of instruction with a caveat that the right balance between traditional and active learning approaches for fundamental courses such as calculus seemed to fit the students best. The results of this study can provide guidelines on how to adopt hands-on activities to increase the learning potential in the current and future generation of students.

ACKNOWLEDGEMENT

This research was partially supported by the SUTD-MIT International Design Centre (IDC). The views and opinions expressed in this article are those of the authors and do not necessarily reflect the views of the IDC or SUTD.

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