1	2	3	4	5	6	7	8	9	10	11	12	**Total**

D o n o t w r i t e a b o v e t h i s l i n e !

YOUR NAME (print):

MATH 114 – 001/002

Sample midterm # 2

November 2012

Your Section/Instructor's Name (circle one): 001/Pop 002/Cooper

Your TA (first name):

Rules:

- A single $8\frac{1}{2}$ by 11 inch handwritten page (one sided) is permitted.
- No other written or printed materials or electronic devices are allowed.

Grading:

- There are 8 problems (with suggested answers) and 4 questions each worth 10 points.
- Do all problems, **showing your work**, and *circling* your answers.
- **This is not a multiple choice exam!** No credit will be given if you circle the right answer, but do not show the work leading to the answer.

Instructions:

- Be prepared to show your Penn ID if asked to do so.
- Write you name at the top of each page of the exam.
- *Do not detach* any of the pages of the exam.

Academic Integrity Statement:

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this Math 114 Midterm Exam.

Name (printed): _____ Signature:

NAME:

1. The point in the plane tangent to the surface $2x^2 + y^2 = z + 3$ at the point $(1, 1, 0)$ which is closest to $(2, 4, 0)$ is:

(a) $(0, 3, 0)$ (b) $(-1, 2, 1)$ (c) $(-1, 0, -1)$ (d) $(0, 0, 0)$ (e) $(-1, -1, 2)$ (f) $(1, 1, 0)$

2. Which of the following planes is parallel to the plane tangent to the graph of the function $z = x^2 + \cos(2yx)$ at the point defined by $(x, y) = (1, \frac{\pi}{2})$?

(a) $-2x + z = 1$ (b) $x + y + 2z = 1$ (c) $-2x - z = 1$

(d) $-x + 2y = 1$ (e) $2x + y + z = 1$ (f) $-2x + y + z = 1$

NAME:

3. The function $\ell(x, y) = (y-1)^2 e^{x+y}$ achieves a local minimum at:

(a) $(0, \frac{1}{2})$ (b) $(0, -\frac{1}{2})$ (c) $(1, 0)$ (d) $(1, 1)$ (e) $(2, 1)$ (f) nowhere

4. One of the tangent planes to the surface $x^2 + 2xy + y^2 + 2x - z + 2 = 0$ which is perpendicular to the yz-plane is:

(a) $2y + z = 0$ (b) $y + z = 0$ (c) $x + z = 0$ (d) $z = 1$ (e) $x + 2y = 0$ (f) $2y = 1$

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5. The distance from the point $(2, 2, 3)$ to the ellipsoid $x^2 + y^2 + 2z^2 = 4$ is:

(a) $\sqrt{6}$ (b) 0 (c) 1 (d) $\sqrt{2}$ (e) $\sqrt{3}$ (f) 2

6. The sum of the absolute maximum and the absolute minimum of the function $f(x, y) = x^2 - 2xy^2 - y^2 - 1$ on the region $\{(x, y) \mid x^2 + 2x + 4y^2 \leq 1\}$ is:

(a) $1 + \sqrt{8}$ (b) $1 + \sqrt{7}$ (c) $1 + \sqrt{6}$ (d) $1 + \sqrt{5}$ (e) $1 + \sqrt{4}$ (f) $1 + \sqrt{3}$

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7. The temperature of a metal plate in the xy-coordinates is given by $T(x, y) = y^2 e^{x-1}$. Suppose that an ant is at the point $(0, 1)$ on the plate and wants to escape from the plate by running in the direction of the fastest temperature decrease. How fast is the temperature decrease at $(0, 1)$ in the direction taken by the ant ?

   a) b) c) d) 0 e) f)

8. Suppose that three positive numbers a, b, c satisfying $\frac{1}{a^2} + \frac{4}{b^2} + \frac{9}{c^2} \leq 3$. Then the minimum of $f(a, b, c) = abc$ is:

(a) 6 (b) 27 (c) 3 (d) 33 (e) 2 (f) 39

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9. The altitude of a hill is described by the function $f(x, y) = 2x^2y - \pi x \cos(y)$, where x is how far east the point is and y is how far north the point is. Is it true that an unrestricted source of water in the point of coordinates $(1, \pi)$ flows due north?

- **Justify your answer!**

10. Let a function $h(x, y)$ have a maximum point at (x_0, y_0). Is it true that the Hessian determinant $h_{xx}(x_0, y_0)\, h_{yy}(x_0, y_0) - h_{xy}(x_0, y_0)^2$ of $h(x, y)$ at (x_0, y_0) must be positive?

- **Justify your answer!**

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11. Find out whether the limit

$$\lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)\big(\cos(xy)-1\big)}{x^4+y^4}$$

exits, and if the limit exists, compute the limit.

12. Is it true that the tangent plane at the surface $z = f(x, y)$ at point of minimum of $f(x, y)$ in the interior of the domain of $f(x, y)$ is parallel to the xy-plane?

- **Justify your answer!**