

Problem set 12

Consider the following functions $f : \mathbb{C} \rightarrow \mathbb{C}$

- (a) $f(z) = \bar{z}$
- (b) $f(z) = \exp(z)$
- (c) $f(z) = \exp(\bar{z})$
- (d) $f(z) = z^2 + 12z\bar{z}^3 + 2$
- (e) $f(z) = |z|$
- (f) $f(z) = |z|^2$
- (g) $f(z) = \frac{z}{|z|}$ for $z \neq 0$; $f(0) = 0$
- (h) $f(z) = \exp\left(-\frac{1}{z^4}\right)$ for $z \neq 0$; $f(0) = 0$
- (i) $f(z) = \frac{z^5}{|z|^4}$ for $z \neq 0$; $f(0) = 0$

For each of the functions (a) - (i), determine the following

1. The set of points at which it is continuous.
2. The set of points at which the given map is \mathbb{R} -differentiable, when viewed as a map $\mathbb{R}^2 \rightarrow \mathbb{R}^2$.
3. The set of points at which the partial derivatives exist, and the set of points at which they satisfy the Cauchy-Riemann equations.
4. The set of points at which f is \mathbb{C} -differentiable
5. The largest open set on which f defines a holomorphic function.

