Recall def'ns of $\mathcal{T}(M)$, $\mathcal{S}$, ℓ_*, I_*, $\pi : [0,\infty)^{\mathcal{S}} \to \mathbb{P}([0,\infty)^{\mathcal{S}})$.

Mod(M) acts on $\mathcal{S}$ in the obvious way: $\varphi \cdot c := \varphi(c)$.

Then $\mathrm{Mod}(M) \curvearrowright [0,\infty)^{\mathcal{S}}$ as: $(\varphi \cdot f)(c) = f(\varphi^{-1} \cdot c)$.

With this, ℓ_* and I_* are equivariant. Let's check ℓ_*.

$$\ell_*(\varphi \cdot (X,m)) = \ell_*((X, m \circ \varphi^{-1})) = \left\{ c \longmapsto \ell_c(X, m \circ \varphi^{-1}) \right\}$$

$$= \left\{ c \longmapsto \ell_{\varphi^{-1}(c)}(X,m) \right\} = \varphi \cdot \ell_*((X,m)).$$

Recall ℓ_* is an embedding. This means a surface is determined by the lengths of its scc's. $\pi \circ \ell_*$ is also an embedding, meaning it's impossible in hyperbolic geometry to simultaneously expand all the scc's by the same factor.

To better understand I_*, let's begin with intersections of scc's.

$$i : \mathcal{S} \times \mathcal{S} \longrightarrow [0,\infty)$$
$$(b, c) \longmapsto i(b,c)$$

is the $\min\limits_{\substack{b' \sim b \\ c' \sim c}} \#|b' \cap c'|$.

$\sim$ means isotopic

FACT: If b' & c' are closed geods in some hyper. metric on M then $i(b,c) = \#\lvert b' \cap c' \rvert$.

With this we can define $i_* : \mathcal{S} \longrightarrow [0,\infty)^{\mathcal{S}}$ & $I_* := \pi \circ i_*$.

$I_*(\mathcal{S}) \subset \mathbb{P}([0,\infty)^{\mathcal{S}})$ has a slightly surprising topology.