

Supplementary exam problems for Chapter 10

- Let $y(t)$ be the solution to $y' = y^2 - t^2$ with initial condition $y(0) = 0$.
 - Use a series solution to evaluate $y(1/2)$.
 - Use Euler iteration to evaluate $y(1/2)$.
 - Which would you use if you need the answer to be correct to $\pm 10^{-6}$?
- Let f be a probability density on the positive reals. Suppose that $f'(x)$ is negative and proportional to square of the probability of picking a number greater than x . Write a differential equation to model this problem.
- Suppose $y(t)$ satisfies $y' = c/y$ for some $c > 0$, where $y(0)$ is some positive number. Like what power does y grow as $t \rightarrow \infty$?
 - Now suppose $y(t)$ satisfies $y' = c/y$ for some $c < 0$, and satisfies the initial condition $y(0) = A$. On what interval $(0, T)$ is there a continuously differentiable solution to this equation?
- A silly model for insect emigration from a hive: let $f(t)$ be the population in hive measure in units of kilograms of biomass. Suppose the emigration rate per unit time is based on overcrowding and model this by saying that $f'(t)$ is negative and proportional to the square of the biomass. Assume a time scale on which emigration is the only source of population change.
 - Write a differential equation to model this problem. Use population units of kilograms of biomass and time units of hours. Be sure to give units for the parameter giving the constant of proportionality.
 - Is there a limiting population as $t \rightarrow \infty$?
 - Find an exact solution for the amount of biomass after t hours.

5. Estimate $\arctan(1.1)$ to three decimal places without a calculator.

6. Find the general solution to

$$y'' = -2x (y')^2.$$

7. Which of the following differential equations have solutions for all initial values $y(0)$ and all times $t \geq 0$?

(a)

$$y' = ye^{-x}$$

(b)

$$y' = y(1 + e^{-x^2})$$

(c)

$$y' = y^2(1 + e^{-x^2})$$

(d)

$$y' = \frac{1 + y^3}{2 + \cos(y)}$$