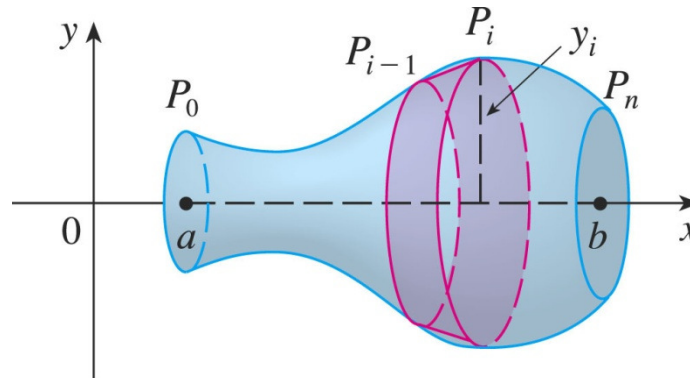


(a) Surface of revolution



(b) Approximating band

$$\begin{aligned} \text{Area of the band} &= 2\pi(\text{radius})(\text{length}) \\ &= 2\pi\left(\frac{y_{i-1} + y_i}{2}\right)d(P_{i-1}P_i) \end{aligned}$$

$$= 2\pi f(x_i^*)\sqrt{1+(f'(x_i^*))^2}\Delta x$$

$$\text{Total surface area} = \sum_{i=1}^n 2\pi f(x_i^*)\sqrt{1+(f'(x_i^*))^2}\Delta x$$

$$\text{Total surface area} = \lim_{n \rightarrow \infty} \sum_{i=1}^n 2\pi f(x_i^*)\sqrt{1+(f'(x_i^*))^2}\Delta x$$

(better approximation)

$$\text{Surface Area} = \int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2} dx$$

For the radius, take an average.

For the length, take the distance from P_{i-1} to P_i .

$$\text{In 9.1 we saw } d(P_{i-1}P_i) = \sqrt{1+(f'(x_i^*))^2}\Delta x$$

$$\text{For small } \Delta x, y_i = f(x_i) \approx f(x_i^*)$$

$$\text{and } y_{i-1} = f(x_{i-1}) \approx f(x_i^*)$$

the area of surface obtained by rotating
the curve $y = f(x)$ about the x -axis for $a \leq x \leq b$ is



A function with a continuous derivative on $[a, b]$



the area of surface obtained by rotating the graph of a function about the **y-axis** for $a \leq x \leq b$ is

$$SA = 2\pi \int_a^b x ds$$

function: $y = f(x)$
$SA = 2\pi \int_a^b x \sqrt{1 + \left[\frac{dy}{dx}\right]^2} dx$

function: $x = g(y) \quad c \leq y \leq d$
$SA = 2\pi \int_c^d g(y) \sqrt{1 + \left[\frac{dx}{dy}\right]^2} dy$

A function with a continuous derivative on $[a, b]$



the area of surface obtained by rotating the graph of a function about the **x-axis** for $a \leq x \leq b$ is

$$SA = 2\pi \int_a^b y ds$$

function: $y = f(x)$
$SA = 2\pi \int_a^b f(x) \sqrt{1 + \left[\frac{dy}{dx}\right]^2} dx$

function: $x = g(y) \quad c \leq y \leq d$
$SA = 2\pi \int_c^d y \sqrt{1 + \left[\frac{dx}{dy}\right]^2} dy$



Find the area of the surface formed by revolving the graph of $f(x) = x^2$ on the interval $0 \leq x \leq \sqrt{2}$ about the y-axis.

$$SA = 2\pi \int_a^b x \sqrt{1 + \left[\frac{dy}{dx} \right]^2} dx$$

$$f(x) = x^2 \Rightarrow \frac{dy}{dx} = 2x$$

$$\sqrt{1 + \left[\frac{dy}{dx} \right]^2} = \sqrt{1 + 4x^2}$$

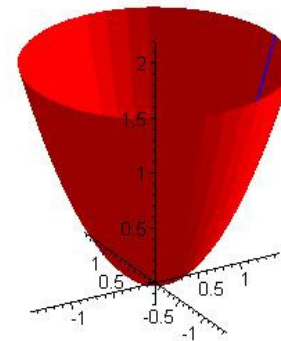
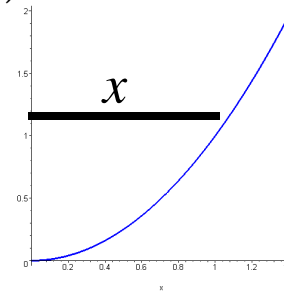
$$SA = 2\pi \int_0^{\sqrt{2}} x \sqrt{1 + 4x^2} dx$$

$$u = 1 + 4x^2$$

$$du = 8x dx \quad \frac{1}{8} du = x dx$$

$$2\pi \frac{1}{8} \int u^{1/2} du = 2\pi \frac{1}{8} \cdot \frac{2}{3} u^{3/2} = \frac{\pi}{6} (1 + 4x^2)^{3/2}$$

$$SA = \frac{\pi}{6} \left[(1 + 4x^2)^{3/2} \right]_0^{\sqrt{2}} = \frac{\pi}{6} [9^{3/2} - 1] = \frac{26\pi}{6} = \boxed{\frac{13\pi}{3}}$$





Find the area of the surface formed by revolving the graph of $x = \frac{1}{9}y^2 + 2$ on the interval $2 \leq y \leq 6$ about the x -axis.

$$SA = 2\pi \int_a^b y \sqrt{1 + \left[\frac{dx}{dy}\right]^2} dy$$

$$g(y) = \frac{1}{9}y^2 + 2 \Rightarrow \frac{dx}{dy} = \frac{2}{9}y$$

$$\sqrt{1 + \left[\frac{dx}{dy}\right]^2} = \sqrt{1 + \frac{4}{81}y^2} = \sqrt{\frac{81 + 4y^2}{81}} = \frac{1}{9}\sqrt{81 + 4y^2}$$

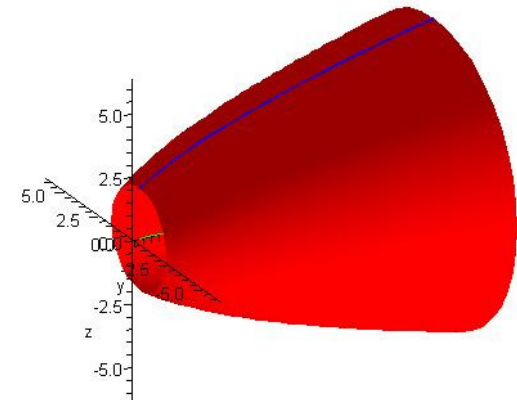
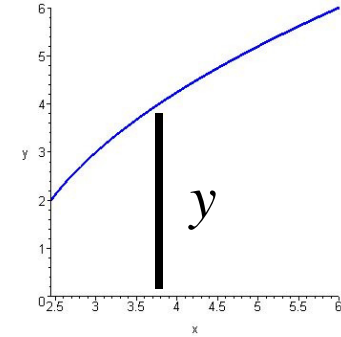
$$SA = \frac{2\pi}{9} \int_2^6 y \sqrt{81 + 4y^2} dy$$

$$u = 81 + 4y^2$$

$$du = 8y dy \quad \frac{1}{8} du = y dy$$

$$\frac{2\pi}{9} \frac{1}{8} \int u^{1/2} du = \frac{2\pi}{9} \frac{1}{8} \cdot \frac{2}{3} u^{3/2} = \frac{\pi}{54} (81 + 4y^2)^{3/2}$$

$$SA = \frac{\pi}{54} \left[(81 + 4y^2)^{3/2} \right]_2^6 = \frac{\pi}{54} \left[225^{3/2} - 97^{3/2} \right]$$





Find the area of the surface formed by revolving the graph of $f(x) = \sqrt{x}$ on the interval $4 \leq x \leq 9$ about the x -axis.

axis: x -axis

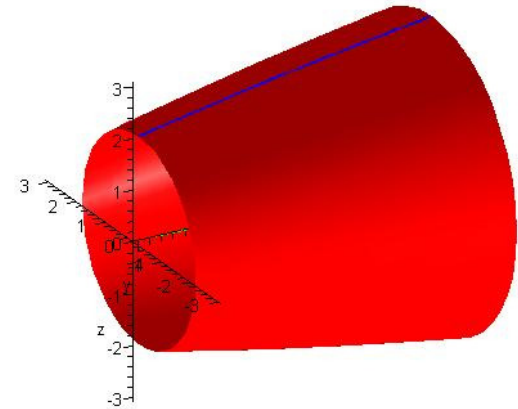
function: $y = f(x)$

$$\Rightarrow SA = 2\pi \int_a^b f(x) \sqrt{1 + \left[\frac{dy}{dx}\right]^2} dx$$

$$f(x) = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\sqrt{1 + \left[\frac{dy}{dx}\right]^2} = \sqrt{1 + \frac{1}{4x}} = \sqrt{\frac{4x+1}{4x}}$$

$$SA = 2\pi \int_4^9 \sqrt{x} \sqrt{\frac{4x+1}{4x}} dx = 2\pi \int_4^9 \sqrt{x \cdot \frac{4x+1}{4x}} dx = \cancel{2}\pi \int_4^9 \frac{1}{\cancel{2}} \sqrt{4x+1} dx = \pi \int_4^9 \sqrt{4x+1} dx$$



$$u = 4x + 1$$

$$du = 4dx \quad \frac{1}{4} du = dx$$

$$\pi \cdot \frac{1}{4} \int u^{1/2} du = \pi \cdot \frac{1}{4} \cdot \frac{2}{3} u^{3/2} = \frac{\pi}{6} (4x+1)^{3/2}$$

$$SA = \frac{\pi}{6} \left[(4x+1)^{3/2} \right]_4^9$$

$$= \frac{\pi}{6} \left[37^{3/2} - 17^{3/2} \right]$$