Name: ________________________ Class: ___________________ Date: __________ ID: A

Leftover 104 Material

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Which of the following equations is satisfied by the function $x = \sin 2t$?

a. $x' - 2t = 0$
b. $x' - 4xt = 0$
c. $x'' + 4x = 0$
d. $x'' + x = 0$
e. $x' - 4x = 0$
f. $x' + 4x = 0$
g. $x'' - 4x = 0$
h. $x'' - x = 0$

____ 2. Which of the following equations is satisfied by the function $x = e^{2t}$?

a. $x' - 2t = 0$
b. $x' - 4xt = 0$
c. $x'' + 4x = 0$
d. $x'' + x = 0$
e. $x' - 4x = 0$
f. $x' + 4x = 0$
g. $x'' - 4x = 0$
h. $x'' - x = 0$

____ 3. Which of the following equations is satisfied by the function $x = e^{2t^2}$?

a. $x' - 2t = 0$
b. $x' - 4xt = 0$
c. $x'' + 4x = 0$
d. $x'' + x = 0$
e. $x' - 4x = 0$
f. $x' + 4x = 0$
g. $x'' - 4x = 0$
h. $x'' - x = 0$

____ 4. Which of the following is a solution of the differential equation $\frac{dy}{dx} + 4y = 0$

a. $y = e^{-4x}$
b. $y = 4x$
c. $y = e^{2x^2}$
d. $y = \sin 2x$
e. $y = e^{2x}$
f. $y = 2x^2$
g. $y = \frac{1}{4x+1}$
h. $y = e^{4x}$

____ 5. Which of the following is a solution of the differential equation $\frac{dy}{dx} - 4y = 0$

a. $y = e^{-4x}$
b. $y = 4x$
c. $y = e^{2x^2}$
d. $y = \sin 2x$
e. $y = e^{2x}$
f. $y = 2x^2$
g. $y = \frac{1}{4x+1}$
h. $y = e^{4x}$

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____ 6. Which of the following is a solution of the differential equation $\frac{dy}{dx}+4y^2=0$

a. $y=e^{-4x}$
b. $y=4x$
c. $y=e^{2x^2}$
d. $y=\sin 2x$
e. $y=e^{2x}$
f. $y=2x^2$
g. $y=\frac{1}{4x+1}$
h. $y=e^{4x}$

____ 7. Which of the following is a solution of the differential equation $\frac{dy}{dx}-4xy=0$

a. $y=e^{-4x}$
b. $y=4x$
c. $y=e^{2x^2}$
d. $y=\sin 2x$
e. $y=e^{2x}$
f. $y=2x^2$
g. $y=\frac{1}{4x+1}$
h. $y=e^{4x}$

____ 8. Which of the following is a solution of the differential equation $\frac{d^2y}{dx^2}+4y=0$

a. $y=e^{-4x}$
b. $y=4x$
c. $y=e^{2x^2}$
d. $y=\sin 2x$
e. $y=e^{2x}$
f. $y=2x^2$
g. $y=\frac{1}{4x+1}$
h. $y=e^{4x}$

____ 9. Which of the following is a solution of the differential equation $\frac{d^2y}{dx^2}-4y=0$

a. $y=e^{-4x}$
b. $y=4x$
c. $y=e^{2x^2}$
d. $y=\sin 2x$
e. $y=e^{2x}$
f. $y=2x^2$
g. $y=\frac{1}{4x+1}$
h. $y=e^{4x}$

____ 10. Which of the following is a solution of the differential equation $\frac{dy}{dx}-4x=0$

a. $y=e^{-4x}$
b. $y=4x$
c. $y=e^{2x^2}$
d. $y=\sin 2x$
e. $y=e^{2x}$
f. $y=2x^2$
g. $y=\frac{1}{4x+1}$
h. $y=e^{4x}$

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____ 11. Which of the following is a solution of the differential equation $\dfrac{dy}{dx} - 4 = 0$

a. $y = e^{-4x}$
b. $y = 4x$
c. $y = e^{2x^2}$
d. $y = \sin 2x$
e. $y = e^{2x}$
f. $y = 2x^2$
g. $y = \dfrac{1}{4x+1}$
h. $y = e^{4x}$

____ 12. Which of the following is a solution of the differential equation $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = 0$

a. $y = e^{-3x}$
b. $y = e^{2x}$
c. $y = xe^{-3x}$
d. $y = xe^{3x}$
e. $y = e^{-2x}$
f. $y = e^{-3x} + x$
g. $y = xe^{3x} + x$
h. None of these

____ 13. Which of the following is a solution of the differential equation $\dfrac{dy}{dx} + 2xy = x$?

a. $y = 1 + e^{-x^2}$
b. $y = e^{2x}$
c. $y = xe^{-x^2}$
d. $y = 1 + e^{-x^2}$
e. $y = \dfrac{1}{2} + e^{-x^2}$
f. $y = e^{-x^2}$
g. $y = \dfrac{1}{2} + e^{x^2}$
h. None of these

____ 14. Which of the following is a solution of the differential equation $\dfrac{d^3y}{dx^3} - \dfrac{d^2y}{dx^2} = 0$ that satisfies the initial conditions $y(0) = -4$, $y'(0) = -2$, $y''(0) = -3$?

a. $y = -1 + x - 3e^{-x}$
b. $y = -1 + x - 3e^{x}$
c. $y = -1 + x + 3e^{x}$
d. $y = -1 + x - 2e^{x}$
e. $y = -1 - 3e^{x}$
f. $y = 1 + x - 4e^{x}$
g. $y = -1 + x^2 - 3e^{x}$
h. None of these

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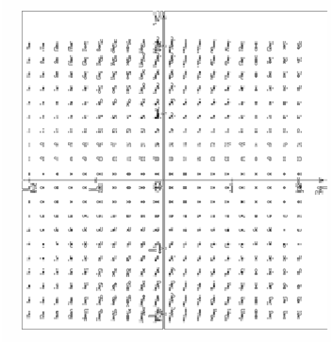
____ 15. A direction field is given below. Which of the following represents its differential equation?



a. $\frac{dy}{dx} = \sin x$

b. $\frac{dy}{dx} = -y$

c. $\frac{dy}{dx} = y - \frac{1}{2}y^2$

d. $\frac{dy}{dx} = x + y$

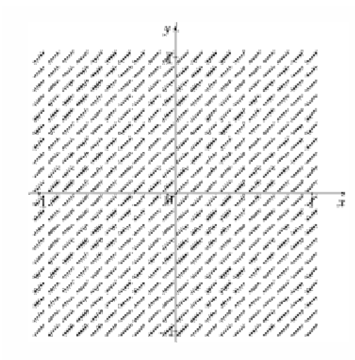
e. $\frac{dy}{dx} = x^2$

f. $\frac{dy}{dx} = 1$

g. $\frac{dy}{dx} = y^2$

h. $\frac{dy}{dx} = x^2 + y^2$

____ 16. A direction field is given below. Which of the following represents its differential equation?



a. $\frac{dy}{dx} = \sin x$

b. $\frac{dy}{dx} = -y$

c. $\frac{dy}{dx} = y - \frac{1}{2}y^2$

d. $\frac{dy}{dx} = x + y$

e. $\frac{dy}{dx} = x^2$

f. $\frac{dy}{dx} = 1$

g. $\frac{dy}{dx} = y^2$

h. $\frac{dy}{dx} = x^2 + y^2$

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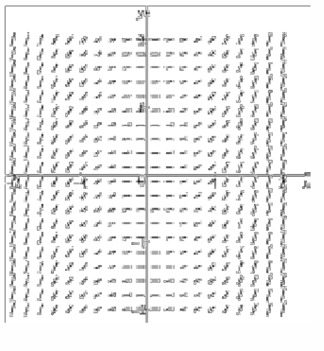
____ 17. A direction field is given below. Which of the following represents its differential equation?



a. $\frac{dy}{dx} = \sin x$

b. $\frac{dy}{dx} = -y$

c. $\frac{dy}{dx} = y - \frac{1}{2}y^2$

d. $\frac{dy}{dx} = x + y$

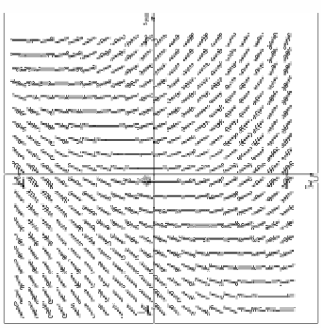
e. $\frac{dy}{dx} = x^2$

f. $\frac{dy}{dx} = 1$

g. $\frac{dy}{dx} = y^2$

h. $\frac{dy}{dx} = x^2 + y^2$

____ 18. A direction field is given below. Which of the following represents its differential equation?



a. $\frac{dy}{dx} = \sin x$

b. $\frac{dy}{dx} = -y$

c. $\frac{dy}{dx} = y - \frac{1}{2}y^2$

d. $\frac{dy}{dx} = x + y$

e. $\frac{dy}{dx} = x^2$

f. $\frac{dy}{dx} = 1$

g. $\frac{dy}{dx} = y^2$

h. $\frac{dy}{dx} = x^2 + y^2$

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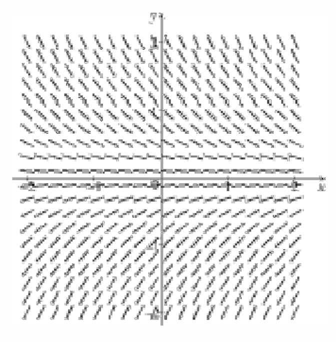
____ 19. A direction field is given below. Which of the following represents its differential equation?



a. $\dfrac{dy}{dx} = \sin x$

b. $\dfrac{dy}{dx} = -y$

c. $\dfrac{dy}{dx} = y - \dfrac{1}{2}y^2$

d. $\dfrac{dy}{dx} = x + y$

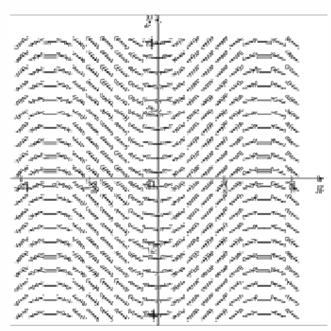
e. $\dfrac{dy}{dx} = x^2$

f. $\dfrac{dy}{dx} = 1$

g. $\dfrac{dy}{dx} = y^2$

h. $\dfrac{dy}{dx} = x^2 + y^2$

____ 20. A direction field is given below. Which of the following represents its differential equation?



a. $\dfrac{dy}{dx} = \sin x$

b. $\dfrac{dy}{dx} = -y$

c. $\dfrac{dy}{dx} = y - \dfrac{1}{2}y^2$

d. $\dfrac{dy}{dx} = x + y$

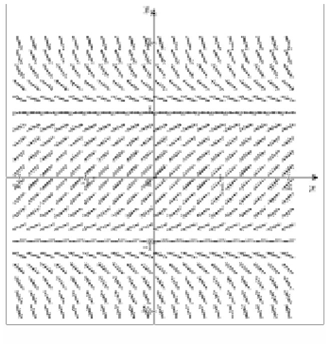
e. $\dfrac{dy}{dx} = x^2$

f. $\dfrac{dy}{dx} = 1$

g. $\dfrac{dy}{dx} = y^2$

h. $\dfrac{dy}{dx} = x^2 + y^2$

____ 21. Which of the given differential equations matches the given direction field?



a. $\frac{dy}{dx} = x - 1$

b. $\frac{dy}{dx} = 1 - y^2$

c. $\frac{dy}{dx} = 1 + y$

d. $\frac{dy}{dx} = x^2 - y^2$

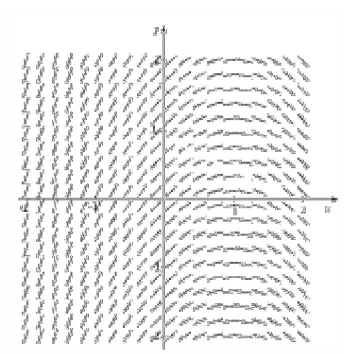
e. $\frac{dy}{dx} = y - 1$

f. $\frac{dy}{dx} = y^2 - x^2$

g. $\frac{dy}{dx} = 1 - x$

h. $\frac{dy}{dx} = y^2 - 1$

____ 22. Which of the given differential equations matches the given direction field?



a. $\frac{dy}{dx} = x - 1$

b. $\frac{dy}{dx} = 1 - y^2$

c. $\frac{dy}{dx} = 1 + y$

d. $\frac{dy}{dx} = x^2 - y^2$

e. $\frac{dy}{dx} = y - 1$

f. $\frac{dy}{dx} = y^2 - x^2$

g. $\frac{dy}{dx} = 1 - x$

h. $\frac{dy}{dx} = y^2 - 1$

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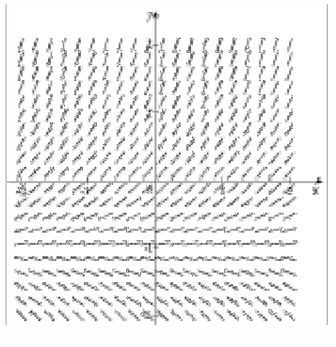
____ 23. Which of the given differential equations matches the given direction field?



a. $\frac{dy}{dx} = x - 1$

b. $\frac{dy}{dx} = 1 - y^2$

c. $\frac{dy}{dx} = 1 + y$

d. $\frac{dy}{dx} = x^2 - y^2$

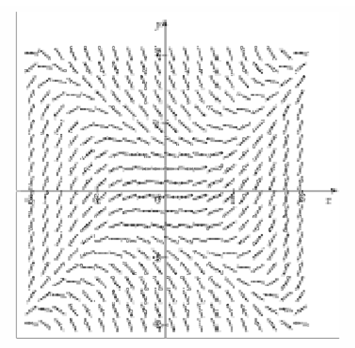
e. $\frac{dy}{dx} = y - 1$

f. $\frac{dy}{dx} = y^2 - x^2$

g. $\frac{dy}{dx} = 1 - x$

h. $\frac{dy}{dx} = y^2 - 1$

____ 24. Which of the given differential equations matches the given direction field?



a. $\frac{dy}{dx} = x - 1$

b. $\frac{dy}{dx} = 1 - y^2$

c. $\frac{dy}{dx} = 1 + y$

d. $\frac{dy}{dx} = x^2 - y^2$

e. $\frac{dy}{dx} = y - 1$

f. $\frac{dy}{dx} = y^2 - x^2$

g. $\frac{dy}{dx} = 1 - x$

h. $\frac{dy}{dx} = y^2 - 1$

____ 25. Suppose $\frac{dy}{dx} = 2x,\ y(0) = 1$. Use Euler's method with step size $h = 2$ to approximate $y(4)$.

a. 5

b. 2

c. 3

d. 17

e. 1

f. 4

g. 9

h. 16

____ 26. Suppose $\frac{dy}{dx} = y^2 - 3$, $y(1) = 2$. Use Euler's method with step size $h = 1$ to approximate $y(3)$.

a. 1
b. 8
c. 16
d. 9
e. 4
f. 14
g. 12
h. 6

____ 27. Suppose $\frac{dy}{dx} = 3y - x$, $y(0) = 1$. Use Euler's method with step size $h = 0.5$ to approximate $y(1)$.

a. 10
b. 7.5
c. 9.5
d. 5
e. 1
f. 2.5
g. 4
h. 6

____ 28. Suppose $\frac{dy}{dx} = y - x^2$, $y(0) = 3$. Use Euler's method with step size $h = 0.2$ to approximate $y(0.4)$.

a. 3.6
b. 3.2
c. 3.4
d. 5.14
e. 1
f. 4.312
g. 4
h. 6.1

____ 29. Suppose $\frac{dy}{dx} = xy^2$, $y(1) = 2$. Use Euler's method with step size $h = 0.5$ to approximate $y(2)$.

a. 28
b. 8
c. 16
d. 3
e. 30
f. 18
g. 15
h. 6

____ 30. Suppose $\frac{dy}{dx} = \frac{y^2}{x}$, $y(1) = 2$. Use Euler's method with step size $h = 1$ to approximate $y(3)$.

a. 24
b. 8
c. 16
d. 4
e. 25
f. 14
g. 12
h. 6

____ 31. Suppose $\frac{dy}{dx} = x + y$, $y(-1) = -2$. Use Euler's method with step size $h = 0.5$ to approximate $y(0)$.

a. 3.5
b. 5.5
c. −3.5
d. −5.5
e. 2.5
f. 6
g. −2.5
h. −6

____ 32. Find the solution of the initial-value problem $\frac{dy}{dx} = x\sin\left(x^2\right)$, $y(0) = 0$.

a. $-\frac{1}{2}\cos\left(x^2\right) - \frac{1}{2}$
b. $-\frac{1}{2}\cos\left(x^2\right)$
c. $-\frac{1}{2}\cos\left(x^2\right) + \frac{1}{2}$
d. $\frac{1}{2}\cos\left(x^2\right) + \frac{1}{2}$
e. $-\frac{1}{2}\sin\left(x^2\right) - \frac{1}{2}$
f. $-\frac{1}{2}\sin\left(x^2\right)$
g. $-\frac{1}{2}\sin\left(x^2\right) + \frac{1}{2}$
h. $\frac{1}{2}\sin\left(x^2\right) + \frac{1}{2}$

____ 33. Solve the differential equation $y' = 5y\left(1000 - y\right)$ subject to the initial condition $y(0) = 500$. From your solution, and the value of the limit $\lim_{t \to \infty} y(t)$.

a. 5000
b. 2500
c. 1000
d. 2000
e. 200
f. 20000
g. 100
h. 500

____ 34. Solve the differential equation $\frac{dy}{dt} = y^2$, $y(0) = 1$. From your solution, and the value of $y(2)$.

a. $\frac{1}{3}$
b. 1
c. $-\frac{1}{3}$
d. -3
e. -1
f. 3
g. $-\frac{1}{5}$
h. $\frac{1}{5}$

____ 35. Solve the differential equation $\frac{dy}{dt} = t\left(y - 3\right)$, $y(2) = 3$. From your solution, and the value of $y(5)$.

a. -2
b. 2
c. 5
d. 0
e. -3
f. 3
g. -5
h. $\frac{1}{5}$

____ 36. Find the solution of the initial-value problem $\frac{dy}{dt} = 2t\sqrt{1 - y}$, $y(1) = 0$.

a. $2\sqrt{y - 1} = 3 - t^2$
b. $\frac{2}{\sqrt{y - 1}} = 3 - t^2$
c. $2\sqrt{1 - y} = 3 + t^2$
d. $\frac{2}{\sqrt{1 - y}} = 3 + t^2$
e. $2\sqrt{1 - y} = -3 + t^2$
f. $\frac{2}{\sqrt{1 - y}} = -3 + t^2$
g. $2\sqrt{1 - y} = 3 - t^2$
h. $\frac{2}{\sqrt{1 - y}} = 3 - t^2$

____ 37. Find the solution of the initial-value problem $y' = \dfrac{\ln x}{xy}$, $y(1) = 2$.

a. $y = \dfrac{1+x}{1+\ln x}$
b. $y = \dfrac{8x}{(1+x)^2}$
c. $y = 2 + 2\ln x$
d. $y = \sqrt{4 + (\ln x)^2}$
e. $y = x\ln x + 2x$
f. $y = x\left(1 + x^2\right)$
g. $y = x + \sqrt{1 + \ln x}$
h. $y = \sqrt{x}\,(1+x)$

____ 38. Find the solution of the initial-value problem $\dfrac{dy}{dt} = 2ty^2 + 3y^2$, $y(0) = 1$.

a. $y = -\dfrac{1}{1 - 3t - t^2}$
b. $y = \dfrac{1}{1 - 3t + t^2}$
c. $y = \dfrac{1}{1 + 3t - t^2}$
d. $y = \dfrac{1}{1 - 3t - t^2}$
e. $y = 1 + 3t - t^2$
f. $y = 1 - 3y - t^2$
g. $y = 1 - 3t + t^2$
h. $y = -1 + 3t + t^2$

____ 39. Solve the initial-value problem $\dfrac{dy}{dt} = 2ty^2 + 3y^2$, $y(0) = 1$. Then use your solution to evaluate $y(1)$.

a. -3
b. 1
c. $-\dfrac{1}{3}$
d. $\dfrac{1}{3}$
e. 0
f. 3
g. $-\dfrac{1}{5}$
h. $\dfrac{1}{5}$

____ 40. Find the solution of the initial-value problem $\dfrac{dy}{dt} = \dfrac{2t}{y^2 + t^2y^2}$, $y(0) = 3$.

a. $y = 3\ln\left(1 + t^2\right) + 3C$
b. $y = \sqrt[3]{3\ln\left(1 + t^2\right) + 3C}$
c. $y = 3\ln\left(1 + t^2\right) + 9$
d. $y = \sqrt[3]{3\ln\left(1 + t^2\right) + 9}$
e. $y = 3\ln\left(1 + t^2\right) + 27$
f. $y = \sqrt[3]{3\ln\left(1 + t^2\right) + 27}$
g. $y = 3\ln\left(1 + t^2\right)$
h. $y = \sqrt[3]{3\ln\left(1 + t^2\right)}$

____ 41. Solve the initial-value problem $\frac{dy}{dt} = \frac{2t}{y^2 + t^2y^2}$, $y(0) = 3$. Then use your solution to evaluate $y\left(\sqrt{e-1}\right)$.

a. $\sqrt[3]{30}$
b. $\sqrt[3]{12}$
c. 30
d. 1
e. $\sqrt[3]{3}$
f. $\sqrt[3]{10}$
g. 12
h. 27

____ 42. Find the solution of the initial-value problem $\frac{dy}{dt} = 3y + 1$, $y(0) = 2$.

a. $y = \frac{1}{3}\left(7e^{t} - 1\right)$
b. $y = \left(7e^{t} - 5\right)$
c. $y = \frac{1}{3}\left(e^{3t} - 1\right)$
d. $y = \left(3e^{3t} - 1\right)$
e. $y = \frac{1}{3}\left(5e^{3t} + 1\right)$
f. $y = \left(5e^{3t} + 1\right)$
g. $y = \frac{1}{3}\left(7e^{3t} - 1\right)$
h. $y = \left(7e^{3t} - 1\right)$

____ 43. Solve the initial-value problem $\frac{dy}{dt} = 3y + 1$, $y(0) = 2$. Then use your solution to evaluate $y(\ln 2)$.

a. $\frac{3}{13}$
b. $\frac{3}{55}$
c. $\frac{7}{3}$
d. $\frac{41}{3}$
e. $\frac{13}{3}$
f. $\frac{55}{3}$
g. $\frac{3}{7}$
h. 9

____ 44. Find the solution of the initial-value problem $\frac{dy}{dt} = \frac{y + t^2y}{t^2}$, $y(1) = 2$.

a. $y = 2e^{t + (1/t)}$
b. $y = 3e^{t - (1/t)}$
c. $y = 2e^{t}$
d. $y = 2e^{1 + (1/t^2)}$
e. $y = 2e^{1/t}$
f. $y = ce^{t - (1/t)}$
g. $y = ce^{t + (1/t)}$
h. $y = 2e^{t - (1/t)}$

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____ 45. Solve the initial-value problem $t\dfrac{dy}{dt} = y\left(y-1\right)$, $y(2) = 4$.

a. $y = -\dfrac{8}{8+3t}$

b. $y^2 - y = 6t$

c. $y = -\dfrac{8}{8-3t}$

d. $y^2 - y = 6t$

e. $y = \dfrac{8}{8-3t}$

f. $y^2 = 8t$

g. $y = \dfrac{1}{1-t}$

h. $y^2 + y = 6t$

____ 46. Which of the following is the power series centered at $a = 0$ for $f(x) = \dfrac{1}{1+4x}$?

1) $\displaystyle\sum_{n=0}^{\infty} (-1)^n 4^n x^n$ 2) $\displaystyle\sum_{n=0}^{\infty} 4^n x^n$ 3) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n} x^n$

a. None

b. 1

c. 2

d. 3

e. 1, 2

f. 1, 3

g. 2, 3

h. 1, 2, 3

____ 47. Which of the following is the power series centered at $a = 0$ for $f(x) = \dfrac{x}{1+4x}$?

1) $\displaystyle\sum_{n=0}^{\infty} (-1)^n 4^n x^n$ 2) $\displaystyle\sum_{n=0}^{\infty} 4^n x^{n+1}$ 3) $\displaystyle\sum_{n=0}^{\infty} (-1)^n 4^n x^{n+1}$

a. None

b. 1

c. 2

d. 3

e. 1, 2

f. 1, 3

g. 2, 3

h. 1, 2, 3

____ 48. Which of the following is the power series centered at $a = 0$ for $f(x) = \dfrac{1}{x+4}$?

1) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n} x^n$ 2) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+1}} x^n$ 3) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4^n} x^{n-1}$

a. None

b. 1

c. 2

d. 3

e. 1, 2

f. 1, 3

g. 2, 3

h. 1, 2, 3

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____ 49. Which of the following is the power series centered at $a = 0$ for $f(x) = \dfrac{1}{1-4x^2}$?

1) $\sum_{n=0}^{\infty} 4^n x^{2n}$ 2) $\sum_{n=0}^{\infty} (2x)^{2n}$ 3) $\sum_{n=0}^{\infty} (-1)^n 4^n x^{2n}$

a. None
b. 1
c. 2
d. 3
e. 1, 2
f. 1, 3
g. 2, 3
h. 1, 2, 3

____ 50. Which of the following is the power series centered at $a = 0$ for $f(x) = \dfrac{1}{x^2+4}$?

1) $\sum_{n=0}^{\infty} \dfrac{(-1)^n}{4^{n+1}} x^n$ 2) $\sum_{n=0}^{\infty} \dfrac{(-1)^n}{4^{n+1}} x^{2n}$ 3) $\sum_{n=0}^{\infty} \dfrac{(-1)^n}{4^n} x^{2n}$

a. None
b. 1
c. 2
d. 3
e. 1, 2
f. 1, 3
g. 2, 3
h. 1, 2, 3

____ 51. Find the coefficient of x^3 in the Maclaurin series for $f(x) = \sin 2x$.

a. $-\dfrac{2}{3}$
b. $-\dfrac{4}{3}$
c. $\dfrac{4}{3}$
d. $-\dfrac{8}{3}$
e. $\dfrac{2}{3}$
f. $\dfrac{8}{3}$
g. $-\dfrac{1}{3}$
h. $\dfrac{1}{3}$

____ 52. Find the radius of convergence of the Maclaurin series for $f(x) = \dfrac{1}{4+x^2}$.

a. 1
b. $\dfrac{1}{8}$
c. ∞
d. $\dfrac{1}{4}$
e. $\dfrac{1}{2}$
f. 4
g. 8
h. 2

____ 53. Find the coefficient of x^4 in the Maclaurin series for $f(x) = e^{-2x}$.

a. 16
b. $-\dfrac{2}{3}$
c. 3
d. 4
e. –16
f. $\dfrac{2}{3}$
g. –3
h. 0

____ 54. Find the coefficient of x^4 in the Maclaurin series for $f(x) = x\cos\left(x^3\right)$.

a. 16
b. $-\frac{2}{3}$
c. 3
d. 4
e. –16
f. $\frac{2}{3}$
g. –3
h. 0

____ 55. Find the coefficient of x^5 in the Maclaurin series for $f(x) = \int \cos\left(x^2\right)dx$.

Note: The series is unique except for the constant of integration.

a. $-\frac{1}{10}$
b. $\frac{1}{15}$
c. $-\frac{1}{5}$
d. $\frac{2}{5}$
e. $-\frac{2}{5}$
f. $-\frac{1}{15}$
g. $\frac{1}{5}$
h. $\frac{1}{10}$

____ 56. Find the terms in the Maclaurin series for the function $f(x) = \ln(1+x)$, as far as the term in x^3.

a. $1 - x + x^2 - x^3$
b. $x - x^2 + x^3$
c. $1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$
d. $x - \frac{1}{2}x^2 + \frac{1}{3}x^3$
e. $1 + \frac{1}{2}x + \frac{2}{3}x^2 + \frac{5}{6}x^3$
f. $x + \frac{1}{2}x^2 + \frac{1}{6}x^3$
g. $1 + \frac{x}{2} + \frac{1}{6}x^2 + \frac{1}{24}x^3$
h. $x - \frac{1}{24}x^2 + \frac{1}{120}x^3$

____ 57. Find the terms in the Maclaurin series for the function $f(x) = e^{-x}$, as far as the term in x^3.

a. $1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$
b. $1 + x + \frac{1}{2}x^2 - \frac{1}{6}x^3$
c. $1 - x + x^2 - x^3$
d. $1 + x + x^2 + x^3$
e. $1 - x + \frac{1}{2}x^2 - \frac{1}{3}x^3$
f. $1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3$
g. $-x + x^3$
h. $x - x^3$

____ 58. Find the first four terms in the Maclaurin series for $f(x) = xe^{-x}$.

a. $x - x^2 + x^3 - x^4$
b. $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$
c. $x - x^2 + \frac{1}{2}x^3 - \frac{1}{6}x^4$
d. $x - 2x^2 + 3x^3 - 4x^4$
e. $x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$
f. $x + x^2 + \frac{1}{3}x^3 + \frac{1}{8}x^4$
g. $x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$
h. $\frac{1}{2}x - \frac{1}{6}x^2 + \frac{1}{24}x^3 - \frac{1}{120}x^4$

____ 59. Find the terms of the Maclaurin series for $\dfrac{1}{\sqrt{1-x}}$, as far as the term in x^3.

a. $1-x+x^2-x^3$
b. $1-\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{8}x^3$
c. $1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{5}{16}x^3$
d. $1-\frac{1}{2}x+\frac{3}{4}x^2-\frac{5}{8}x^3$
e. $1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{6}x^3$
f. $1-\frac{1}{2}x+\frac{1}{6}x^2+\frac{1}{24}x^3$
g. $1+\frac{1}{2}x+\frac{3}{4}x^2+\frac{15}{16}x^3$
h. $1-\frac{1}{2}x+\frac{3}{8}x^2+\frac{7}{24}x^3$

____ 60. Find the terms of the Maclaurin series for $f(x)=\dfrac{1}{\sqrt{1+2x}}$, as far as the term in x^3.

a. $1-x+x^2-x^3$
b. $1+x-\frac{1}{2}x^2+\frac{1}{3}x^3$
c. $1-x+\frac{3}{2}x^2-\frac{5}{2}x^3$
d. $1+x+3x^2+5x^3$
e. $1-x+\frac{3}{2}x^2-\frac{7}{3}x^3$
f. $1+x+\frac{1}{2}x^2+\frac{7}{3}x^3$
g. $1-x+\frac{5}{2}x^2-\frac{7}{3}x^3$
h. $1+x+\frac{7}{2}x^2+\frac{11}{3}x^3$

____ 61. Find the coefficient of x^3 in the binomial series for $(1+x)^5$.

a. 3
b. 6
c. 15
d. 20
e. 10
f. 5
g. 16
h. 12

____ 62. Find the coefficient of x in the binomial series for $\sqrt{1+x}$.

a. 2
b. –1
c. 1
d. $-\frac{1}{2}$
e. $\frac{1}{2}$
f. $-\sqrt{2}$
g. –2
h. $\sqrt{2}$

____ 63. Find the coefficient of x^3 in the binomial series for $\sqrt{1+x}$.

a. $-\frac{1}{2}$
b. $-\frac{1}{4}$
c. $\frac{1}{8}$
d. $\frac{1}{2}$
e. $-\frac{1}{8}$
f. $-\frac{1}{16}$
g. $\frac{1}{16}$
h. $\frac{1}{4}$

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____ 64. Use the binomial series to expand the function $\sqrt{4+x}$ as a power series. Give the coefficient of x^2 in that series.

a. $-\frac{1}{8}$
b. $-\frac{1}{32}$
c. $-\frac{1}{64}$
d. $\frac{1}{8}$
e. $\frac{1}{32}$
f. $-\frac{1}{16}$
g. $\frac{1}{64}$
h. $\frac{1}{16}$

____ 65. How many coefficients in the binomial series expansion of $(1+x)^7$ are divisible by 7?

a. 0
b. 5
c. 7
d. 3
e. 2
f. 6
g. 1
h. 4

____ 66. Find the coefficient of x^3 in the binomial series for $\frac{1}{(1+x)^4}$.

a. 6
b. 20
c. –6
d. –10
e. –20
f. –12
g. 10
h. 12

____ 67. Which of the following is the degree 4 Taylor polynomial centered at $a = 0$ for $f(x) = \cos(2x)$?

1) $1-(2x)^2+16x^4$ 2) $1-2x^2+\frac{2}{3}x^4$ 3) $1-\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}$

a. None
b. 1
c. 2
d. 3
e. 1, 2
f. 1, 3
g. 2, 3
h. 1, 2, 3

____ 68. Which of the following is the degree 3 Taylor polynomial centered at $a = 0$ for $f(x) = \cos(2x)$?

1) $1-(2x)^2+16x^4$ 2) $1-2x^2+\frac{8}{3}x^4$ 3) $1-\frac{(2x)^2}{2!}$

a. None
b. 1
c. 2
d. 3
e. 1, 2
f. 1, 3
g. 2, 3
h. 1, 2, 3

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____ 69. Which of the following is the degree 3 Taylor polynomial centered at $a = 0$ for $f(x) = \frac{1-\cos x}{x}$?

1) $\frac{x}{2} - \frac{x^3}{24}$ 2) $-\frac{x}{2} + \frac{x^3}{24}$ 3) $x - x^3$

a. None
b. 1
c. 2
d. 3
e. 1, 2
f. 1, 3
g. 2, 3
h. 1, 2, 3

____ 70. Which of the following is the degree 2 Taylor polynomial centered at $a = 2$ for $f(x) = \ln x$?

1) $\ln 2 - \frac{x-2}{2} - \frac{(x-2)^2}{8}$ 2) $\ln 2 + \frac{x-2}{2} - \frac{(x-2)^2}{4}$ 3) $\ln 2 + \frac{x-2}{2} - \frac{(x-2)^2}{8}$

a. None
b. 1
c. 2
d. 3
e. 1, 2
f. 1, 3
g. 2, 3
h. 1, 2, 3

____ 71. Which of the following is the degree 2 Taylor polynomial centered at $a = 3$ for $f(x) = \ln x$?

1) $\ln 3 + \frac{x-3}{3} - \frac{(x-3)^2}{18}$ 2) $\ln 3 + \frac{x-3}{3} - \frac{(x-3)^2}{9}$ 3) $\ln 3 - \frac{x-3}{3} + \frac{(x-3)^2}{18}$

a. None
b. 1
c. 2
d. 3
e. 1, 2
f. 1, 3
g. 2, 3
h. 1, 2, 3

____ 72. Which of the following is the degree 2 Taylor polynomial centered at $a = -1$ for $f(x) = \frac{1}{x}$?

1) $-1 - x - x^2$ 2) $-1 - (x+1) - (x+1)^2$ 3) $-1 - (x+1) - 2(x+1)^2$

a. None
b. 1
c. 2
d. 3
e. 1, 2
f. 1, 3
g. 2, 3
h. 1, 2, 3

____ 73. According to Taylor's Formula, what is the maximum error possible in the use of the sum $\sum_{n=0}^{4} \frac{x^n}{n!}$ to approximate e^x in the interval $-1 \le x \le 1$?

a. $\frac{e}{240}$
b. $\frac{e}{48}$
c. $\frac{e}{480}$
d. $\frac{e}{24}$
e. $\frac{e}{20}$
f. $\frac{e}{120}$
g. $\frac{e}{12}$
h. $\frac{e}{60}$

____ 74. Find the coefficient of $(x-2)^2$ in the Taylor polynomial $T_2(x)$ for the function x^3 at the number 2.

a. 3
b. 0
c. 1
d. 6
e. 2
f. 5
g. 8
h. 4

____ 75. What is the smallest value of n that will guarantee (according to Taylor's Formula) that the Taylor polynomial T_n at the number 0 will be within 0.0001 of e^x for $0 \le x \le 1$?

a. 4
b. 5
c. 8
d. 6
e. 7
f. 2
g. 3
h. 9

____ 76. Estimate the range of values of x for which the approximation $\frac{1}{x} = 1-(x-1)+(x-1)^2$ is accurate to within 0.01.

a. [0.68, 1.41]
b. [0.61, 1.54]
c. [0.995, 1.005]
d. [1.51, 2.59]
e. [0.95, 1.05]
f. [0.80, 1.23]
g. [0.980, 1.023]
h. [0.89, 1.14]

____ 77. Estimate the range of values of x for which the approximation $\ln x = \ln 2 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2$ is accurate to within 0.01.

a. [1.08, 3.20]
b. [1.80, 2.20]
c. [0.89, 3.56]
d. [1.43, 2.66]
e. [0.45, 1.78]
f. [0.71, 1.33]
g. [1.90, 2.10]
h. [1.99, 2.01]

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Short Answer

78. (a) Show that every member of the family of functions $y = ce^{\frac{1}{3}x^3}$ is a solution of the differential equation $y' = x^2 y$.

(b) Find a solution of the differential equation $y' = x^2 y$ that satisfies the initial condition $y(0) = 8$.

(c) Find a solution of the differential equation $y' = x^2 y$ that satisfies the initial condition $y(8) = 0$.

79. (a) What can you conclude about the functions which satisfy $y' = y^2$ just by looking at the differential equation?

(b) Verify that $y = -\dfrac{1}{x+c}$ are solutions of the equation in part (a).

(c) Is there a solution of the equation in part (a) that is not a member of the family of functions in part (b)? Justify your answer.

(d) Find a solution to the equation in part (a) with the additional condition that $y(0) = \dfrac{1}{3}$.

80. (a) What can you conclude about the graph of the solution of the equation $\dfrac{dy}{dx} = -\dfrac{x}{y}$ just by looking at the differential equation?

(b) Use implicit differentiation to verify all members of the family $x^2 + y^2 = c^2$ are solutions of the equation in part (a).

(c) Find a solution of the equation in part (a) with the additional condition that $y(3) = 4$.

81. A function $y(x)$ satisfies the differential equation $\dfrac{dy}{dx} = y^2 - 4y + 3$.

(a) What are the constant solutions of the equation?

(b) For what values of y is y increasing?

(c) The equation shows that $\dfrac{dy}{dx}$ is independent of x. Using this observation, what can you say about the relationship of the graphs of the non-constant solutions of the equation?

82. The study of free fall provides one context to consider differential equations. In the simplest case, in the absence of air or other resistance, physicists assume that the rate of change of velocity of a body is constant.

(a) Write an equation for $\frac{dv}{dt}$.

(b) As the time t increases without bound, what happens to the velocity v?

83. The study of free fall provides one context to consider differential equations. In the simplest case, in the absence of air or other resistance, physicists assume that the rate of change of velocity of a body is constant. But it is more realistic to consider the presence of air resistance. Assume that g is the constant of acceleration due to earth's gravity. Suppose that air resistance is proportional to the velocity of the falling body.

(a) Explain why the differential equation $\frac{dv}{dt} = g - kv$, where k is a positive constant, would be a reasonable model for velocity under these conditions.

(b) When does v increase most rapidly? Justify your answer.

(c) Consider the equation in part (a). What would happen to the rate of change of velocity, $\frac{dv}{dt}$, as t increases? Justify your conclusion.

(d) Make a sketch of a possible solution for this differential equation.

84. In paleontology, the phenomenon of radioactive decay is commonly used to date fossil remains. This method uses the fact that the rate of decay $\frac{dA}{dt}$ of an element will be proportional to the amount A of the radioactive element that exists at time t.

(a) Determine a differential equation that $A(t)$ must satisfy.

(b) What can you say about $A(t)$ as $t \to \infty$?

(c) Find a solution to your differential equation.

(d) How long will it take for half of the original amount of the radioactive element to decay?

85. In biology, it is often assumed that the number of people that will contract a certain disease is directly proportional to the product of the number of people y who are currently infected with the disease at a particular time t and the number of people x who, at the same time, are not yet infected but who are susceptible to it. Assume that the total population, P, in the study is a constant.

(a) What does $\frac{dy}{dt}$ mean?

(b) Determine a differential equation that $y(t)$ must satisfy.

(c) What happens to y as $t \to \infty$?

86. A ball is thrown directly upward from the ground with an initial speed 24 ft/s.

(a) Determine the differential equation that the position function $y(t)$ satisfies.

(b) What are the initial conditions?

(c) How high does the ball rise?

87. The brakes of a car traveling 60 mph decelerate the car at the rate of 20 ft/s^2.

(a) Determine the differential equation that the position function $y(t)$ satisfies.

(b) What are the initial conditions?

(c) If the car is 175 feet from a barrier when the brakes are applied, will it hit the barrier?

88. A tank is filled with 100 L (liter) of water. Brine containing 0.4 kg of salt per liter is poured into the tank at 5 L per minute, and the well-stirred mixture leaves the tank at the same rate.

(a) Write the differential equation that the amount of salt, $s(t)$ in the tank must satisfy.

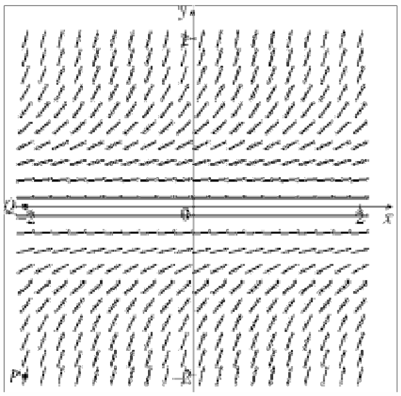
(b) Find the amount of salt in the tank at any time, t.

(c) How much salt is present after 20 min?

(d) What is the concentration of solution in the tank after a long time?

89. Suppose a hail ball melts at a rate proportional to its surface area. If from $t = 0$ to $t = 5$ minutes its radius decreases from $r = 3$ cm to $r = 2$ cm, how long does it take to melt completely?

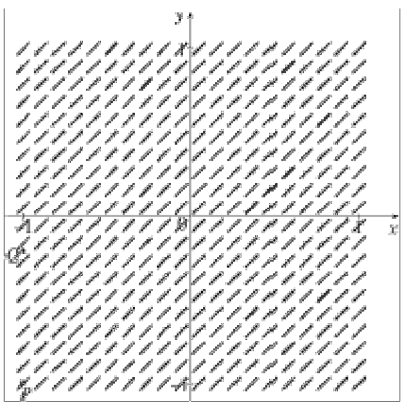
90. A direction field for a differential equation is given below:



(a) Sketch the graphs of the solutions that have initial condition P and initial condition Q.

(b) Determine whether the differential equation is autonomous. Explain your answer.

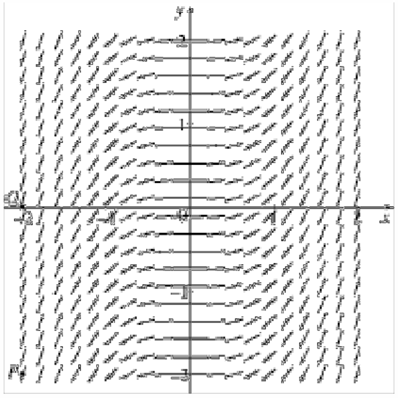
91. A direction field for a differential equation is given below:



(a) Sketch the graphs of the solutions that have initial condition P and initial condition Q.

(b) Determine whether the differential equation is autonomous. Explain your answer.

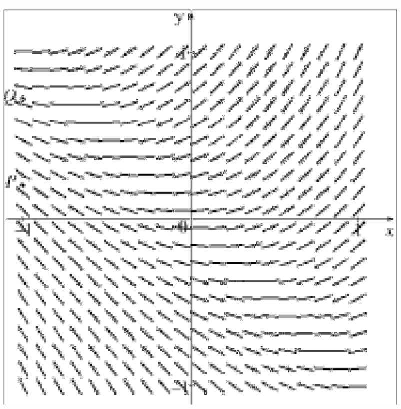
92. A direction field for a differential equation is given below:



(a) Sketch the graphs of the solutions that have initial condition P and initial condition Q.

(b) Determine whether the differential equation is autonomous. Explain your answer.

93. A direction field for a differential equation is given below:



(a) Sketch the graphs of the solutions that have initial condition P and initial condition Q.

(b) Determine whether the differential equation is autonomous. Explain your answer.

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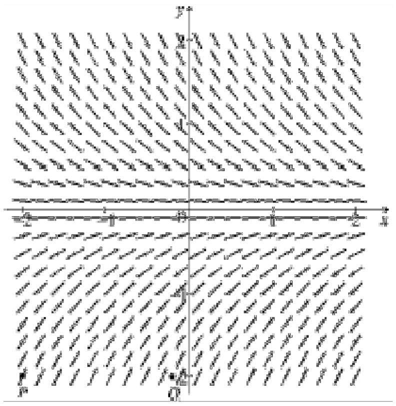
94. A direction field for a differential equation is given below:



(a) Sketch the graphs of the solutions that have initial condition P and initial condition Q.

(b) Determine whether the differential equation is autonomous. Explain your answer.

95. A population is modeled by the differential equation $\frac{dP}{dt} = 0.4P\left(1 - \frac{P}{250}\right)$ where $P(t)$ is the population at time t.

(a) What are the equilibrium solutions?

(b) For what values of P is the population increasing?

(c) For what values of P is the population decreasing?

(d) Use the information from above to sketch the direction field for the given differential equation.

96. Consider the differential equation $\frac{dy}{dt} = y^3 - 2y^2 - 15y$.

(a) What are the equilibrium solutions?

(b) For what values of y is $y(t)$ increasing?

(c) For what values of y is $y(t)$ decreasing?

(d) Use the information from above to sketch the direction field for the given differential equation.

97. Consider the differential equation $\frac{dy}{dt} = t + 1$.

(a) What are the equilibrium solutions?

(b) Sketch the direction field for the given differential equation.

98. Consider the differential equation $\frac{dy}{dt} = t^2 - 1$.

(a) What are the equilibrium solutions?

(b) Sketch the direction field for the given differential equation.

99. Consider the differential equation $\frac{dy}{dt} = y - t$.

(a) What are the equilibrium solutions?

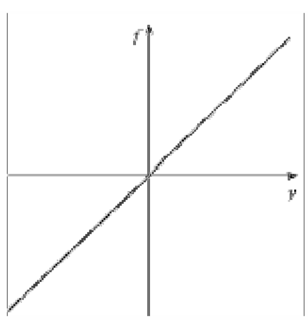
(b) What are the points in the *ty*-plane at which the slope of the solution curve is 0?

(c) What are the points in the *ty*-plane at which the slope of the solution curve is 1?

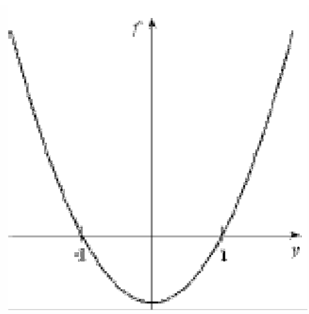
(d) What are the points in the *ty*-plane at which the slope of the solution curve is –1?

(e) Use the information from above to sketch the direction field for the given differential equation.

100. Make a rough sketch of a directional field for the autonomous differential equation $y' = f(y)$ where f is the function graphed below.



101. Make a rough sketch of a directional field for the autonomous differential equation $y' = f(y)$ where f is the function graphed below.



102. (a) Determine the solution of the differential equation $\frac{dy}{dx} = -y$ where $y(0) = 1$.

(b) Use the solution $y(x)$ from part (a) to calculate $y(0.4)$.

(c) Use Euler's Method with the given step sizes to estimate the value of $y(0.4)$ for the equation given in part (a).
(i) $h = 0.4$
(ii) $h = 0.2$
(iii) $h = 0.1$

(d) Sketch $y(x)$ from part (b) and each of the Euler approximations from part (c) on the same set of axes.

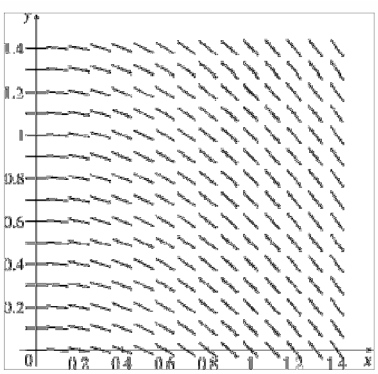
103. (a) Determine the solution of the differential equation $\frac{dy}{dx} = 2y$ where $y(0) = 2$.

(b) Use the solution $y(x)$ from part (a) to calculate $y(0.4)$.

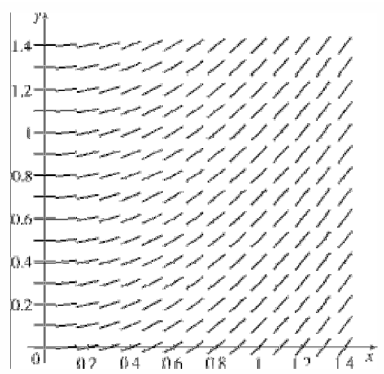
(c) Use Euler's Method with the given step sizes to estimate the value of $y(0.4)$ for the equation given in part (a).
(i) $h = 0.4$
(ii) $h = 0.2$
(iii) $h = 0.1$

(d) Sketch $y(x)$ from part (b) and each of the Euler approximations from part (c) on the same coordinate plane.

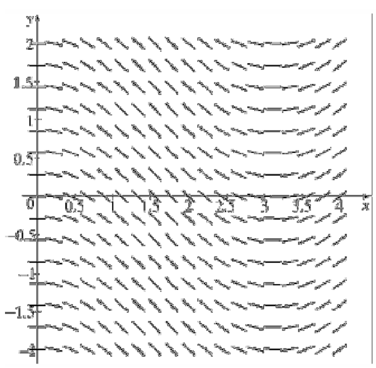
104. A direction field for a differential equation is given below. Use a straightedge to draw the graphs of the Euler approximations to the solution curve over the interval $[0, 1.2]$ that passes through $y(0) = 1$. Use as step sizes $h = 1.2$, $h = 0.6$, and $h = 0.2$.



105. A direction field for a differential equation is given below. Use a straightedge to draw the graphs of the Euler approximations to the solution curve over the interval $[0, 1.2]$ that passes through $y(0) = 0.1$. Use as step sizes $h = 1.2$, $h = 0.6$, and $h = 0.2$.



106. A direction field for a differential equation is given below. Use a straightedge to draw the graphs of the Euler approximations to the solution curve over the interval $[0, 4]$ that passes through $y(0) = 1$. Use as step sizes $h = 4$, $h = 2$, $h = 1$ and $h = 0.5$.



107. Consider the differential equation $\frac{dy}{dx} = 2x + y$.

(a) Sketch the direction field. Indicate where the slopes are -1, 0, or 1. Draw some other slopes as well.

(b) If the point $(1, 2)$ is on the graph of a solution, use Euler's Method with step size 0.5 to estimate the value of the solution at $x = 2.5$.

108. Consider the differential equation $x\frac{dy}{dx} - 3y = 0$.

(a) Sketch the direction field. Indicate where the slopes are -1, 0, or 1. Draw some other slopes as well.

(b) If the point $(1, 2)$ is on the graph of a solution, use Euler's Method with step size 0.5 to estimate the value of the solution at $x = 2$.

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109. Consider the differential equation $t^2 \frac{dy}{dt} = \sqrt{y}$.

(a) Find the general solution to the differential equation.

(b) Find the solution with the initial-value $y(1) = 0$.

(c) Find the solution with the initial-value $y(1) = 1$.

110. Consider the differential equation $\frac{dy}{dt} = \frac{2tx + x}{t^2 + t}$.

(a) Find the general solution to the differential equation.

(b) Find the solution that satisfies the initial condition $x(1) = 2$.

111. Find the equation of a curve that passes through the point $(0, 7)$ and whose slope at a point (x, y) is $4x^3 y$.

112. Consider the differential equation $x^2 y' = y^3$.

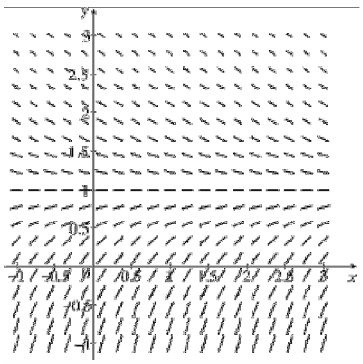
(a) Find the general solution to the differential equation.

(b) Find the solution that satisfies the initial condition $y(1) = 1$.

113. Find the solution to the differential equation $xy' = \sqrt{1 - y^2}$, $x > 0$ that satisfies the initial condition $y(1) = 1$.

114. Find the solution to the differential equation $\frac{dy}{dx} = e^{x-y}$ that satisfies the initial condition $y(0) = 1$.

115. The graph of a direction field for the differential equation $\frac{dy}{dx} = 1 - y$ is given below:



(a) Sketch a solution curve that satisfies the given condition, but without solving the differential equation:
(i) $y(0) = 1$
(ii) $y(2) = 1$
(iii) $y(0) = 0$

(b) Solve the differential equation for each of the conditions in part (a). Compare your answers to the curves you produced in part (a).

(c) What is the relationship between the curves (i) and (ii) in part (a)? Explain why this occurs.

116. Find the orthogonal trajectories of the family of curves $xy = k$. Then draw several members of each family on the same coordinate plane.

117. Find the orthogonal trajectories of the family of curves $x^2 + y^2 = k$. Then draw several members of each family on the same coordinate plane.

118. Find the orthogonal trajectories of the family of curves $y = kx^2$. Then draw several members of each family on the same coordinate plane.

119. In the presence of air resistance for an object in free fall, the velocity is the solution of the differential equation $\frac{dv}{dt} = g - kv$ where g is the constant of acceleration due to earth's gravity and k is a positive constant.

(a) Find the solution for the equation that satisfies the initial value that $v(0) = 0$.

(b) As time, t, increases without bound, what is the limiting velocity. (Note: the limiting velocity during free fall is called the terminal velocity.)

(c) Sketch your solution from part (a).

120. In a model of epidemics, let $y(t)$, in thousands, be the number of infected individuals in the population at time t, in days. If we assume that the infection spreads to all those who are susceptible, one possible solution for $y(t)$ is given by $\frac{dy}{dt} = k\left(P - y\right) \cdot y$ where k is a positive constant which measures the rate of infection and P, in thousands, is the total population in this situation.

(a) Determine the solution of this differential equation if $y(0) = 1$.

(b) Discuss what $y(0) = 1$ means.

(c) As the time increases without bound, what happens to y? (That is, what does $\lim_{t \to \infty} y$ mean?)

(d) Sketch the solution of the differential equation in part (a).

121. A model of the seasonal changes in daylight hours x is given by $\frac{dx}{dt} = kx\cos(wt)$, where k and w are constants.

(a) Determine the general solution for this differential equation.

(b) Discuss the practical meaning of the solution of this equation.

122. A tank is filled with 200 gallons of brine in which is dissolved 5 pounds of salt. Brine containing 0.1 pound of salt per gallon enters the tank at a rate of 2 gallons per minute, and the well-stirred mixture is drawn from the talk at the same rate.

(a) Find the amount of salt in the tank at time t.

(b) How much salt is present in the tank after 20 minutes?

(c) How much salt is present after a long time? What is the concentration then?

123. A tank contains 10 gallons of water and 4 pounds of a chemical, Z, per gallon. To decrease the concentration of Z, pure water is added to the container at a rate of 2 gallons per minute, and the well stirred mixture is drawn from the tank at the same rate.

(a) Find the amount of chemical Z in the tank at time t.

(b) How much chemical Z is present after 10 minutes?

(c) How long does it take for the concentration of chemical Z to be reduced to 0.1 pound per gallon?

124. A tank contains 500 liters of brine with 10 kg of dissolved salt. Pure water enters the tank at a rate of 10 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank:

(a) after t minutes?

(b) after 10 minutes?

125. A tank contains 500 liters of brine with 10 kg of dissolved salt. Pure water enters the tank at a rate of 10 liters per minute. The well-mixed solution drains from the tank 8 liters per minute. Determine the differential equation that the amount of salt in the tank at time t must satisfy.

126. A tank contains 1000 liters of pure water. Brine that contains 0.05 kg of salt per liter of water enters the tank at a rate of 8 liters per minute. Brine that contains 0.04 kg of salt per liter of water enters the tank at a rate of 5 liters per minute. The well-mixed solution drains from the tank at a rate of 13 liters per minute. How much salt is in the tank after:

(a) t minutes?

(b) half an hour?

(c) a very long time?

127. The discharge value on a 1000 liter tank that is filled with water is opened at time $t = 0$ and the water flows out at a rate of 10 liters per second. At the same time a 1% chlorine mixture enters the tank at a rate of 6 liters per second. Assuming that the solution is well-mixed throughout the tank, what is the concentration of chlorine when the tank is half full?

128. Newton's Law of Cooling states that the rate at which a body changes temperature is proportional to the difference between its temperature and the temperature of the surrounding medium. Suppose that a body has an initial temperature of 250 °F and that after one hour the temperature is 200 °F. Assuming that the surrounding air is kept at a constant temperature of 72 °F, determine the temperature of the body at time t.

129. According to Newton's Law of Cooling, the temperature T of a warm object decreases at a rate proportional to the difference between T and the temperature T_o of its surroundings.

(a) Write down this law as a differential equation.

(b) Assume the room temperature is 70 °F. If it takes 2 minutes for a cup of hot coffee to cool down to 180 °F, and how long it takes to cool a cup of coffee from 200 °F to 100 °F.

130. According to Newton's Law of Heating, the temperature T of a cold object increases at a rate proportional to the difference between T and the temperature T_o of its surroundings.

(a) Write down this law as a differential equation.

(b) Assume the room temperature is 70 °F. If it takes 10 minutes for a can of soda to warm up from 30 °F to 35 °F, and how long it takes to warm up a can of soda from 30 °F to 40 °F.

131. Find a power series representation for the function $f(x) = \dfrac{1-x}{1+x}$ and give its interval of convergence.

132. Find a power series representation for the function $f(x) = \dfrac{x}{x+5}$ and find its interval of convergence.

133. Find a power series representation for the function $f(x) = \ln\sqrt{1-x}$ and determine the radius of convergence.

134. Find a power series representation for the function $f(x) = \arctan(3x)$ and determine its radius of convergence.

135. Evaluate $\int \dfrac{1}{1+x^5}\,dx$ as a power series.

136. Find a power series representation for $\ln\left(1+x^2\right)$.

Hint: What is $\dfrac{d}{dx}\ln\left(1+x^2\right)$?

137. Find a power series representation for $\dfrac{1}{(1-x)^2}$ and give its radius of convergence.

138. Approximate the definite integral $\int_0^1 e^{-x^2}\,dx$ accurate to six decimal places.

139. Show that $f(x) = \sum_{n=0}^{\infty} \dfrac{(8x)^n}{n!}$ is a solution to the differential equation $\dfrac{dy}{dx} = 8y$.

140. Find the sum of the series $\sum_{n=2}^{\infty} (n-1)x^n$, where $|x| < 1$.

141. Find a power series representation for the function $f(x) = \dfrac{x^2}{1-3x}$.

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142. Approximate the definite integral $\int_0^{0.1} \frac{1}{x^5+1}\,dx$ accurate to six decimal places.

143. Approximate the definite integral $\int_0^{0.5} \frac{\ln(1+x)}{x}\,dx$ accurate to six decimal places.

144. Find the sum of the series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$.

145. Given that the power series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$, find the power series for $\frac{1}{(1-x)^2}$ in terms of powers of x.

146. Find the power series for $f(x) = \frac{5x+7}{x^2+2x-3}$ in terms of powers of x.
Hint: Express $f(x)$ in terms of partial fractions.

147. Find a power series representation for $\int \frac{x}{x^3+1}\,dx$.

148. Give the Taylor series expansion of $f(x) = \sin x$ about the point $c = \frac{\pi}{4}$.

149. Find the Taylor series for $y = \ln x$ at 2.

150. Find the Taylor polynomial of degree 4 at 0 for the function defined by $f(x) = \ln(1+x)$. Then compute the value of $\ln(1.1)$ accurate to as many decimal places as the polynomial of degree 4 allows.

151. Find the Maclaurin series expansion for $f(x) = \ln(1-x)$ and determine the interval of convergence.

152. If the Maclaurin series for $f(x)$ is $1 - 9x + 16x^2 - 25x^3 + \cdots$, find $f^3(0)$.

153. Find the Taylor series for $x\cos x$ about the origin.

154. Express $f(x) = \int_0^x \frac{\sin t}{t}\,dt$ as a Maclaurin Series.

155. Express $\int_0^x \cos\sqrt{t}\,dt$ as a Maclaurin Series.

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156. Express $\int_0^x \frac{t}{1-t^5}\,dt$ as a Maclaurin Series.

157. (a) Express $\int_0^x \sin(t^2)\,dt$ as a Maclaurin series.

(b) Evaluate $\int_0^{0.3} \sin(x^2)\,dx$ as a series.

158. (a) Express $\int_0^x \frac{\sin(t^2)}{t^2}\,dt$ as a Maclaurin series.

(b) Evaluate $\int_0^{0.3} \frac{\sin(x^2)}{x^2}\,dx$ as a series.

159. Find the Maclaurin series expansion with $n = 5$ for $f(x) = 2^x$. Use this expansion to approximate $2^{0.1}$.

160. Find the coefficient of x^2 in the Maclaurin series for $f(x) = 1/(x+2)$.

161. Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{3^{2n}(2n+1)!}$.

162. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$.

163. Find the sum of the series $\sum_{n=1}^{\infty} \frac{n3^n}{4^n}$.

164. Find the sum of the series $\sum_{n=0}^{\infty} \frac{1}{(2n)!}$.

165. If $f(x) = e^{x^2}$, compute $f^{(11)}(0)$.

166. If $f(x) = e^{x^2}$, compute $f^{(10)}(0)$.

167. Let $f(x) = \frac{x}{1+\frac{1}{4}x^2}$, compute $f^{(23)}(0)$.

168. Use the binomial series to expand $\dfrac{1}{(1+x)^3}$ as a power series. State the radius of convergence.

169. Use the binomial series to expand $\sqrt[3]{1+x^2}$ as a power series. State the radius of convergence.

170. Use the binomial series to expand $\dfrac{x^2}{\sqrt{1-x^3}}$ as a power series. State the radius of convergence.

171. Use the binomial series to expand $\sqrt[5]{x-1}$ as a power series. State the radius of convergence.

172. Use the binomial series formula to obtain the Maclaurin series for $f(x) = (1+x)^{1/3}$.

173. Use the binomial series to expand $\dfrac{1}{\sqrt{2+x}}$ as a power series. State the radius of convergence.

174. Use the binomial series to expand $(4+x)^{3/2}$ as a power series. State the radius of convergence.

175. Find the terms in the power series expansion for the function $f(x) = \dfrac{1}{\sqrt{1+x^2}}$, as far as the term in x^3.

176. Find the terms of the Maclaurin series for $\dfrac{1}{\sqrt{1+2x}}$, as far as the term in x^3.

177. Estimate the range of values of x for which the approximation $\sqrt{x^2+3} = 2 + \dfrac{1}{2}(x-1) + \dfrac{3}{16}(x-1)^2$ is accurate to within 0.0002.

178. Estimate the range of values of x for which the approximation $e^x \cos x = 1 + x - \dfrac{1}{3}x^3$ is accurate to within 0.001.

179. Find the second-degree Taylor polynomial of the function $f(x) = xe^x$ at $a = -1$.

180. Find the Taylor polynomial $T_3(x)$ for the function $f(x) = \dfrac{5x}{2+4x}$ at the point $x_0 = 0$.

181. Consider the function $f(x) = 3x^4 - 24x^3 + 72x^2 - 96x + 49$.

(a) Find the fourth-degree Taylor polynomial of f at $a = 2$.

(b) What is the remainder?

(c) What is the absolute minimum value of f, and where does it occur?

182. Write the fourth-degree Taylor polynomial centered about the origin for the function $f(x) = e^{-2x}$.

183. Find the second-degree Taylor polynomial for $f(x) = \sqrt{x}$, centered about $a = 100$. Also obtain a bound for the error in using this polynomial to approximate $\sqrt{100.1}$.

184. Find an approximation for $\sin(0.1)$ accurate to 6 decimal places.
(*Note:* sin's argument is measured in radians.)

185. Give the 4th-degree Taylor polynomial for $f(x) = \sqrt{x}$ about the point $x = 4$. Using this polynomial, approximate $\sqrt{4.2}$. Give the maximum error for this approximation.

186. Use the 3rd-degree Taylor polynomial of $f(x) = \sqrt{x}$ about $x = 4$ to approximate $\sqrt{6}$. Use the remainder term to give an upper bound for the error in this approximation.

187. Write the Taylor polynomial at 0 of degree 4 for $f(x) = \ln(1+x)$.

188. The first three derivatives of $f(x) = (x+4)^{3/2}$ are $f'(x) = \dfrac{3(x+4)^{1/2}}{2}$, $f''(x) = \dfrac{3}{4(x+4)^{1/2}}$ and

$f'''(x) = \dfrac{-3}{8(x+4)^{3/2}}$.

(a) Give the first four terms of the Taylor series associated with f at $a = -3$.

(b) Give the second-order Taylor polynomial, $T_2(x)$, associated with f at $a = 0$.

(c) Suppose that $x \ge 0$ and that $T_2(x)$ from part (b) is used to approximate $f(x)$. Prove that the error in this approximation does not exceed $\dfrac{x^3}{128}$.

189. Given $f(x) = \tan x$ and $a = 0$,

(a) calculate $T_1(x;\,0)$. (b) calculate $T_3(x;\,0)$. (c) calculate $T_5(x;\,0)$.

190. Find the third Taylor polynomial associated with $f(x) = 2 + x^2 - 5x^3$ at $a = 0$. What is the remainder?

191. (a) Find the third-order Taylor polynomial associated with $f(x) = \sin^{-1} x$ at $a = 0$.

(b) Use the Taylor polynomial from part (a) to find an approximation of $\sin^{-1} 0.2$.

(c) Compare the value you calculated in part (b) with your calculator's value for $\sin^{-1} 0.2$

192. (a) Use series to compute $\int_0^1 x \cos x \, dx$ correct to three decimal places.

(b) Use integration by parts to compute $\int_0^1 x \cos x \, dx$.

(c) Compare your answers in parts (a) and (b) above.

193. Use series to compute $\int_0^1 \cos \sqrt{x} \, dx$ correct to four decimal places.

194. Find the third-degree Taylor polynomial centered at $x = 1$ for $f(x) = \ln x$. Use this result to approximate $\ln 1.2$.

195. Find the third-degree Taylor polynomial of the function $f(x) = \int_0^x \sin t^2 \, dt$ at $a = 0$.

196. Find the third-degree Taylor polynomial of the function $f(x) = e^{x^2} - x^2 \cos \sqrt{x}$ at $a = 0$.

197. Find the second-degree Taylor polynomial of the function $f(x) = x \ln x$ at $a = \dfrac{1}{e}$.

Leftover 104 Material
Answer Section

MULTIPLE CHOICE

1.	ANS: C	PTS: 1
2.	ANS: G	PTS: 1
3.	ANS: B	PTS: 1
4.	ANS: A	PTS: 1
5.	ANS: H	PTS: 1
6.	ANS: G	PTS: 1
7.	ANS: C	PTS: 1
8.	ANS: D	PTS: 1
9.	ANS: E	PTS: 1
10.	ANS: F	PTS: 1
11.	ANS: B	PTS: 1
12.	ANS: D	PTS: 1
13.	ANS: E	PTS: 1
14.	ANS: B	PTS: 1
15.	ANS: G	PTS: 1
16.	ANS: F	PTS: 1
17.	ANS: E	PTS: 1
18.	ANS: D	PTS: 1
19.	ANS: B	PTS: 1
20.	ANS: A	PTS: 1
21.	ANS: B	PTS: 1
22.	ANS: G	PTS: 1
23.	ANS: C	PTS: 1
24.	ANS: D	PTS: 1
25.	ANS: G	PTS: 1
26.	ANS: D	PTS: 1
27.	ANS: H	PTS: 1
28.	ANS: F	PTS: 1
29.	ANS: C	PTS: 1
30.	ANS: A	PTS: 1
31.	ANS: D	PTS: 1
32.	ANS: C	PTS: 1
33.	ANS: C	PTS: 1
34.	ANS: E	PTS: 1
35.	ANS: F	PTS: 1
36.	ANS: G	PTS: 1
37.	ANS: D	PTS: 1
38.	ANS: D	PTS: 1
39.	ANS: C	PTS: 1

40.	ANS: F	PTS:	1
41.	ANS: A	PTS:	1
42.	ANS: G	PTS:	1
43.	ANS: F	PTS:	1
44.	ANS: H	PTS:	1
45.	ANS: E	PTS:	1
46.	ANS: B	PTS:	1
47.	ANS: D	PTS:	1
48.	ANS: G	PTS:	1
49.	ANS: E	PTS:	1
50.	ANS: C	PTS:	1
51.	ANS: B	PTS:	1
52.	ANS: H	PTS:	1
53.	ANS: F	PTS:	1
54.	ANS: H	PTS:	1
55.	ANS: A	PTS:	1
56.	ANS: D	PTS:	1
57.	ANS: A	PTS:	1
58.	ANS: C	PTS:	1
59.	ANS: C	PTS:	1
60.	ANS: C	PTS:	1
61.	ANS: E	PTS:	1
62.	ANS: E	PTS:	1
63.	ANS: G	PTS:	1
64.	ANS: C	PTS:	1
65.	ANS: F	PTS:	1
66.	ANS: E	PTS:	1
67.	ANS: G	PTS:	1
68.	ANS: D	PTS:	1
69.	ANS: B	PTS:	1
70.	ANS: D	PTS:	1
71.	ANS: B	PTS:	1
72.	ANS: C	PTS:	1
73.	ANS: F	PTS:	1
74.	ANS: D	PTS:	1
75.	ANS: E	PTS:	1
76.	ANS: F	PTS:	1
77.	ANS: D	PTS:	1

SHORT ANSWER

78. ANS:

(a) $y' = \left(ce^{\frac{x^3}{3}}\right)' = cx^2e^{\frac{x^3}{3}} = x^2\left(ce^{\frac{x^3}{3}}\right) = x^2y$

(b) $y = 8e^{\frac{x^3}{3}}$

(c) $y = 0$

PTS: 1

79. ANS:

(a) $y = 0$ is a constant solution and if $y \neq 0$, $\frac{dy}{dx} > 0$. Therefore solutions of the differential equation are increasing functions.

(b) $y' = \frac{1}{(x+c)^2} = \left(-\frac{1}{x+c}\right)^2 = y^2$

(c) $y = 0$ is not included in part (b).

(d) $y = \frac{-1}{x-3} = \frac{1}{3-x}$

PTS: 1

80. ANS:

(a) Let $\left(x, y(x)\right)$ be a graph of a solution of the differential equation. Then $\frac{dy}{dx} \cdot \frac{y}{x} = -\frac{x}{y} \cdot \frac{y}{x} = -1$, so the slope of the tangent of $y(x)$ is perpendicular to the line passing through $\left(0,0\right)$ and $\left(x, y(x)\right)$.

(b) $2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

(c) $x^2 + y^2 = 25$

PTS: 1

81. ANS:

(a) $y = 1$ and $y = 3$.

(b) For $y < 1$ or $y > 3$, $y(x)$ is increasing.

(c) The slopes corresponding to two different points with the same y-coordinates must be equal. This means that if we know one solution to the equation, then we can obtain infinitely many others just by shifting the graph of the known solution to the right or left.

PTS: 1

82. ANS:

(a) $\frac{dv}{dt} = a$ where a is a constant.

(b) Since $v(t) = at + v_o$, where v_o is a constant, therefore as t increases without bound, $v(t)$ will increase without bound.

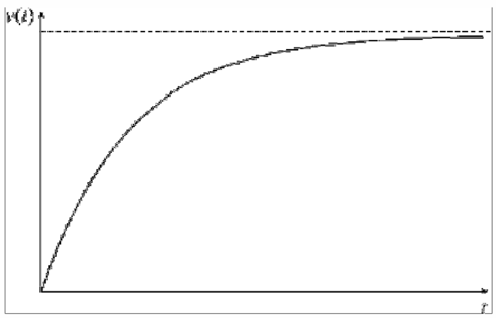
PTS: 1

83. ANS:

(a) $\frac{dv}{dt} = g - kv$ shows that the ratio at which the velocity increases, decreases at a rate proportional to the velocity.

(b) $v'(0) = g > g - kv$ for $v > 0$. Therefore $v(t)$ increases most rapidly at $t = 0$.

(c) As t increases, $\frac{dv}{dt}$ will approach 0. That means the force of resistance is just equal to the gravity. Therefore, the velocity will approach a limiting value.

(d) 

PTS: 1

84. ANS:

(a) $\frac{dA}{dt} = -kA$, where k is a positive constant.

(b) Since $\frac{dA}{dt} < 0$, $A(t)$ will approach 0 as $t \to \infty$.

(c) $A(t) = ce^{-kt}$

(d) $t = \frac{\ln 2}{k}$

PTS: 1

85. ANS:

(a) $\frac{dy}{dt}$ is the infection rate.

(b) $\frac{dy}{dt} = ky\left(P - y\right)$

(c) Since $\frac{dy}{dt} > 0$, as $t \to \infty$, $y(t) \to P$.

PTS: 1

86. ANS:
(a) $y''(t) = -32$
(b) $v(0) = 24,\ y(0) = 0$
(c) 9 feet

PTS: 1

87. ANS:
(a) $y''(t) = -20\text{ ft/s}^2$
(b) $v(0) = 88\text{ ft/s},\ y(0) = 0$
(c) Yes, because the care needs 193.6 feet in order to stop.

PTS: 1

88. ANS:
(a) $s'(t) = 0.4 * 5 - \dfrac{5s(t)}{100},\ s(0) = 0$
(b) $s(t) = 40 - 40e^{-0.05t}$
(c) $s(20) = 40 - 40e^{-1}$
(d) $\lim\limits_{t\to\infty} \dfrac{s(t)}{100} = 0.4$ kg per liter

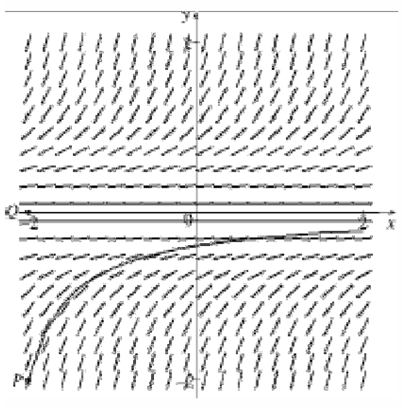
PTS: 1

89. ANS:
$\dfrac{dV}{dt} = -kA$ where V represents the volume of the hail ball and A represents the surface area of the ball.
We know that $V = \dfrac{4}{3}\pi r^3 \Rightarrow \dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt} = A\dfrac{dr}{dt} = -kA \Rightarrow \dfrac{dr}{dt} = -k \Rightarrow r = -kt + c$
$t = 0,\ r = 3 \Rightarrow c = 3$ and $t = 5,\ r = 2 \Rightarrow k = 0.2 \Rightarrow r(t) = -0.2t + 3$
The hail ball melted completely, $\Rightarrow r = 0 \Rightarrow t = 15$ minutes.

PTS: 1

90. ANS:

(a)

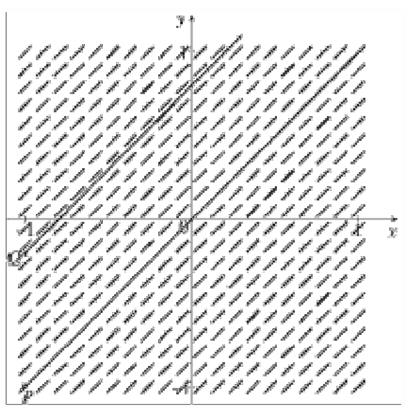


(b) It is autonomous since $\frac{dy}{dx}$ is independent of x.

PTS: 1

91. ANS:

(a)

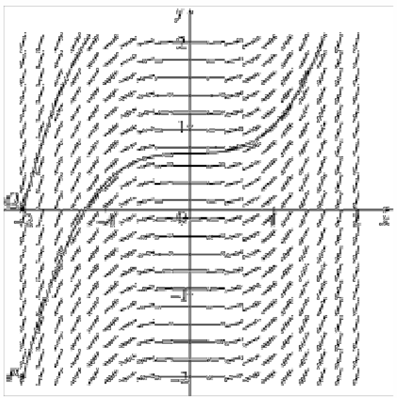


(b) It is autonomous since $\frac{dy}{dx}$ is constant and independent of x.

PTS: 1

92. ANS:

(a)

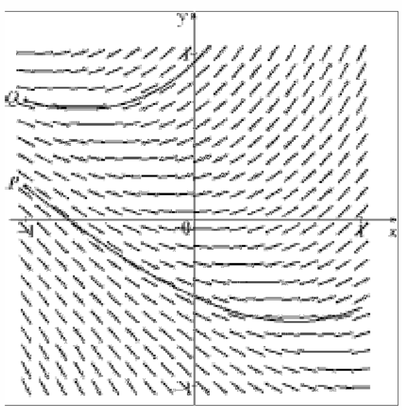


(b) It is not autonomous since $\frac{dy}{dx}$ is not independent of x.

PTS: 1

93. ANS:

(a)

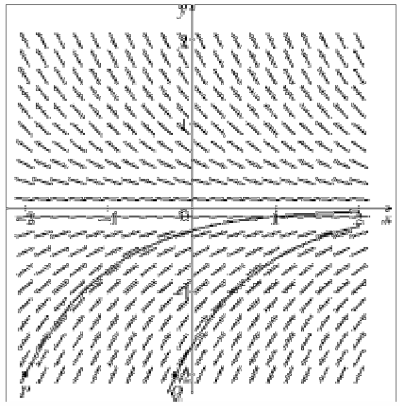


(b) It is not autonomous since $\frac{dy}{dx}$ is dependent on both x and y.

PTS: 1

94. ANS:

(a)



(b) It is autonomous since $\frac{dy}{dx}$ is independent of x.

PTS: 1

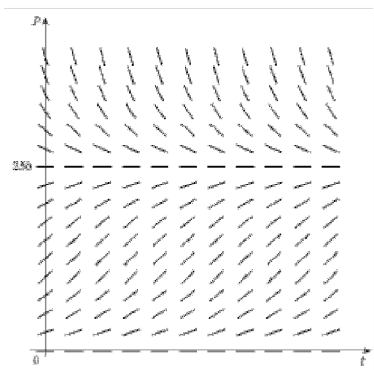
95. ANS:

(a) $P = 0$, $P = 250$

(b) $0 < P < 250$

(c) $0 < P < 250$

(d)



PTS: 1

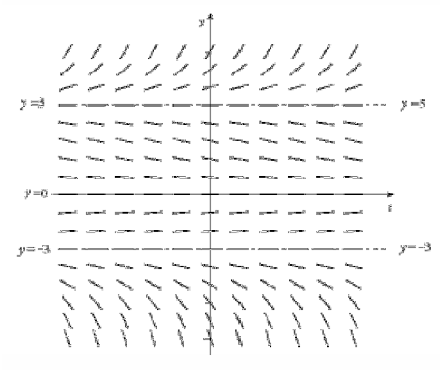
96. ANS:

(a) $y = 0,\ y = 3,\ y = 5$

(b) $-3 < y < 0$ or $y > 5$

(c) $y < -3$ or $0 < y < 5$

(d)

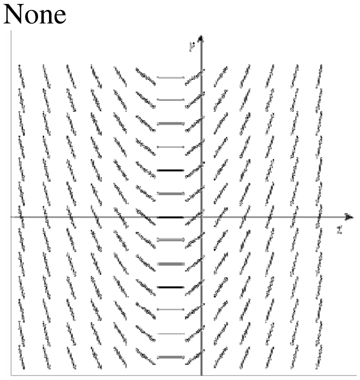


PTS: 1

97. ANS:

(a) None

(b)

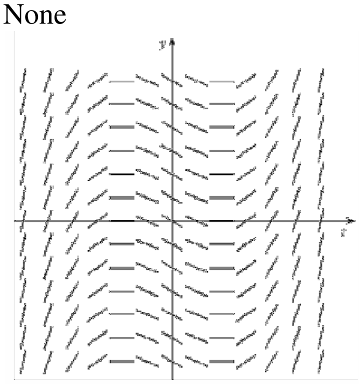


PTS: 1

98. ANS:

(a) None

(b)



PTS: 1

99. ANS:

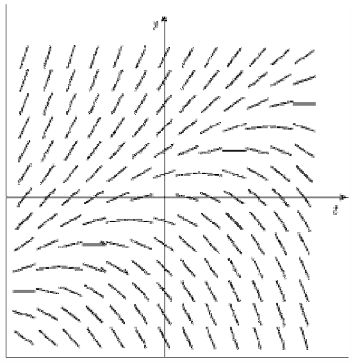
(a) None

(b) $\{(t,t) \mid t \in \mathbb{R}\}$

(c) $\{(t,t+1) \mid t \in \mathbb{R}\}$

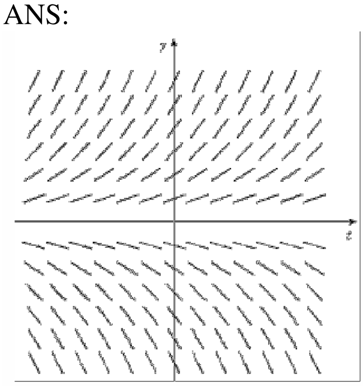
(d) $\{(t,t-1) \mid t \in \mathbb{R}\}$

(e)



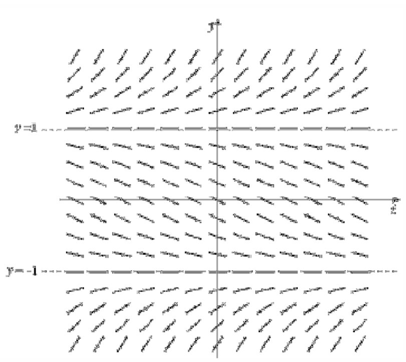
PTS: 1

100. ANS:



PTS: 1

101. ANS:



PTS: 1

102. ANS:

(a) $y = e^{-x}$

(b) $y(0.4) = e^{-0.4} \approx 0.67032$

(c) (i) $y(0) = 1$ and $y'(0) = -y = -1 \Rightarrow y = -x + 1 \Rightarrow y(0.4) = 0.6$

(ii) $y(0) = 1,\, y'(0) = -1 \Rightarrow y = -x + 1 \Rightarrow y(0.2) = 0.8,$

$y'(0.2) = -y(0.2) = -0.8 \Rightarrow y = -0.8x + 0.96 \Rightarrow y(0.4) = 0.64$

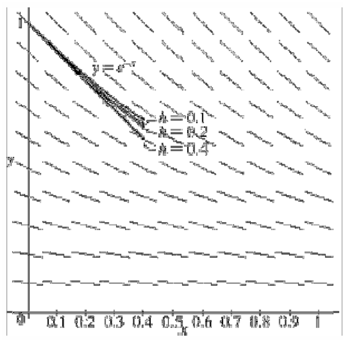
(iii) $y(0) = 1,\, y'(0) = -1 \Rightarrow y = -x + 1 \Rightarrow y(0.1) = 0.9,$

$y'(0.1) = -y(0.1) = -0.9 \Rightarrow y = -0.9x + 0.99 \Rightarrow y(0.2) = 0.81,$

$y'(0.2) = -0.81 \Rightarrow y = -0.81x + 0.972 \Rightarrow y(0.3) = 0.729,$

$y'(0.3) = -0.729 \Rightarrow y = -0.729x + 0.9477 \Rightarrow y(0.4) = 0.6561$

(d)



PTS: 1

103. ANS:

(a) $y(x) = 2e^{2x}$

(b) $y(0.4) = 2e^{2 \cdot 0.4} = 2e^{0.8} \approx 4.4510819$

(c) (i) $y(0) = 2$ and $y'(0) = 4 \Rightarrow y = 4x + 2 \Rightarrow y(0.4) = 3.6$

(ii) $y(0) = 2$ and $y'(0) = 4 \Rightarrow y = 4x + 2 \Rightarrow y(0.2) = 2.8,$

$y'(0.2) = 5.6 \Rightarrow y = 5.6x + 1.68 \Rightarrow y(0.4) = 3.92$

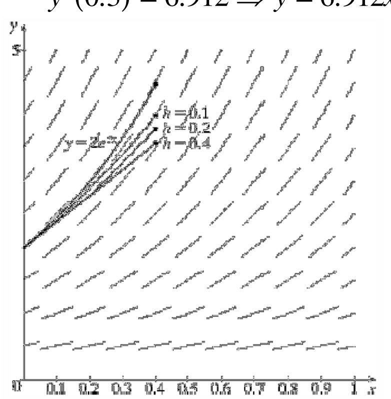
(iii) $y(0) = 2$ and $y'(0) = 4 \Rightarrow y = 4x + 2 \Rightarrow y(0.1) = 2.4,$

$y'(0.1) = 4.8 \Rightarrow y = 4.8x + 1.92 \Rightarrow y(0.2) = 2.88,$

$y'(0.2) = 5.67 \Rightarrow y = 5.67x + 1.728 \Rightarrow y(0.3) = 3.456,$

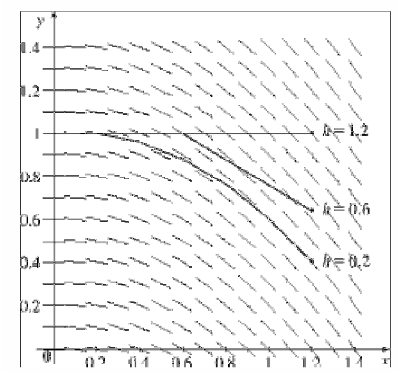
$y'(0.3) = 6.912 \Rightarrow y = 6.912x + 1.3824 \Rightarrow y(0.4) = 4.1472$

(d)



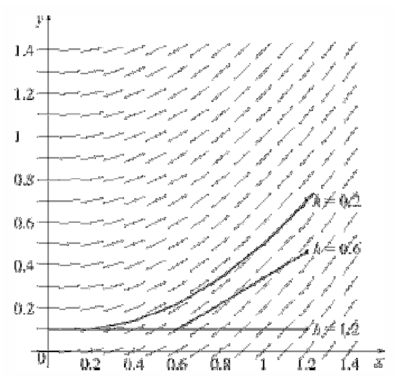
PTS: 1

104. ANS:



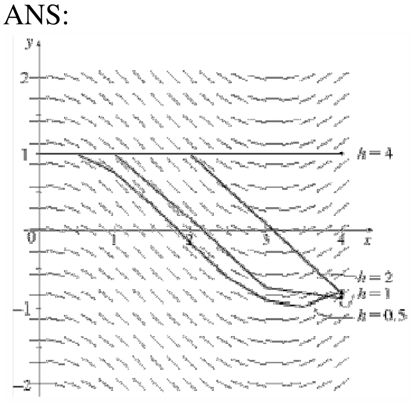
PTS: 1

105. ANS:



PTS: 1

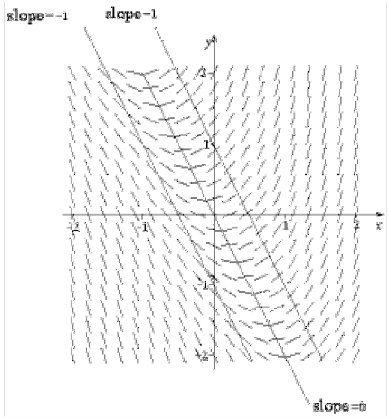
106. ANS:



PTS: 1

107. ANS:

(a)

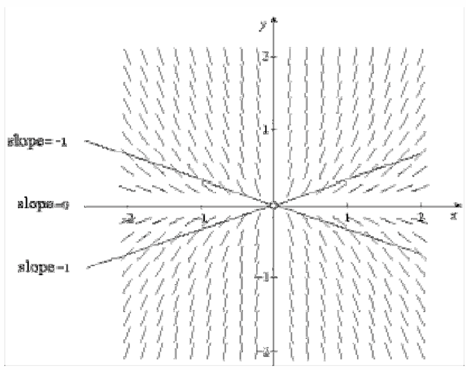


(b) $y = f(2.5) \approx 13.25$

PTS: 1

108. ANS:

(a)



(b) $y = f(2) \approx 10$

PTS: 1

109. ANS:

(a) $y = 0,\ 2\sqrt{y} = -\dfrac{1}{t} + C$

(b) $y = 0$

(c) $2\sqrt{y} = -\dfrac{1}{t} + 3$

PTS: 1

110. ANS:

(a) $x = c\left(t^2 + t\right)$

(b) $x = t^2 + t$

PTS: 1

111. ANS:

$y = 7e^{x^4}$

PTS: 1

112. ANS:

(a) $y = 0$ or $y^2 = \dfrac{x}{2 + cx}$

(b) $y = \sqrt{\dfrac{x}{2 - x}}$

PTS: 1

113. ANS:

$y = \sin\left(\ln x + \dfrac{\pi}{2}\right)$

PTS: 1

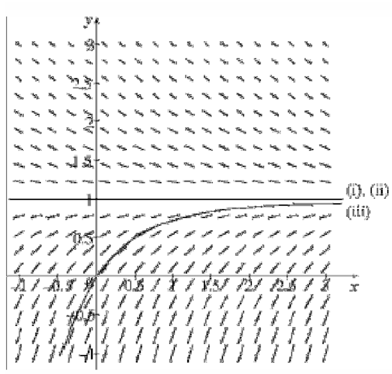
114. ANS:

$y = \ln\left(e^x + e - 1\right)$

PTS: 1

115. ANS:

(a)



(i) $y(0) = 1$ (ii) $y(2) = 1$ (ii) $y(0) = 0$

(b) $y = 1; y = 1; y = 1 - e^{-x}$

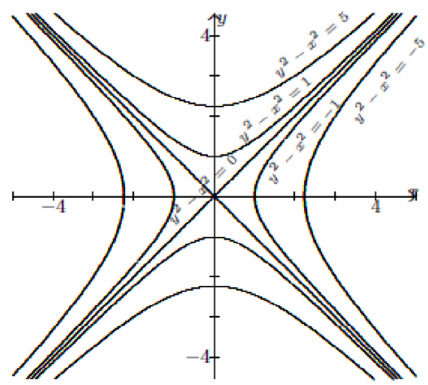
(c) The solutions to (i) and (ii) are the same.

PTS: 1

116. ANS:

$xy = k \Rightarrow x\dfrac{dy}{dx} + y = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{y}{x} \Rightarrow$ slope field for orthogonal trajectories is $\dfrac{dy}{dx} = \dfrac{x}{y} \Rightarrow$

solve the equation $y\,dy = x\,dx \Rightarrow y^2 - x^2 = c$.

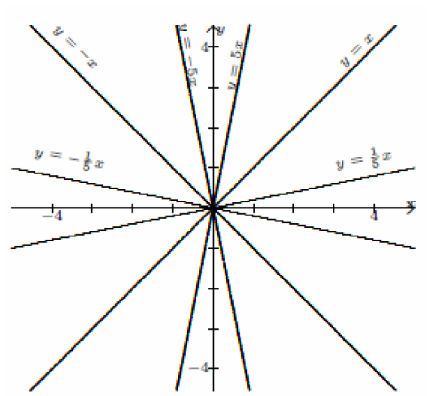


PTS: 1

117. ANS:

$x^2 + y^2 = k \Rightarrow 2x + 2y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{x}{y} \Rightarrow$ slope field for orthogonal trajectories is

$\dfrac{dy}{dx} = \dfrac{y}{x} \Rightarrow$ solve the equation $\dfrac{1}{y}\ dy = \dfrac{1}{x}\ dx \Rightarrow y = kx.$

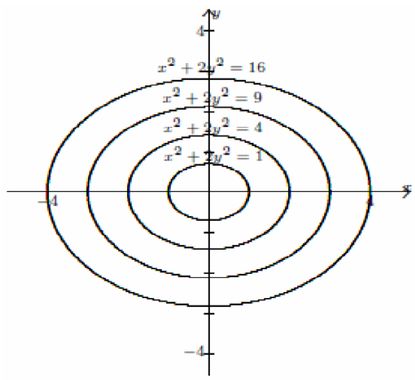


PTS: 1

118. ANS:

$y = kx^2 \Rightarrow \dfrac{dy}{dx} = 2xk \Rightarrow \dfrac{dy}{dx} = 2x\dfrac{y}{x^2} = \dfrac{2y}{x} \Rightarrow$ differential equation for orthogonal trajectories: $\dfrac{dy}{dx} = -\dfrac{x}{2y}$

$2y\,dy = -x\,dx \Rightarrow \dfrac{x^2}{2} + y^2 = k$ or $x^2 + 2y^2 = a^2$



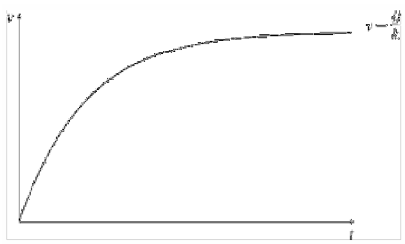
PTS: 1

119. ANS:

(a) $\dfrac{dv}{dt} = g - kv \Rightarrow \dfrac{dv}{g - kv} = dt \Rightarrow v(t) = \dfrac{g}{k}\left(1 - e^{-kt}\right)$

(b) $\lim\limits_{t \to \infty} v(t) = \dfrac{g}{k}$

(c)



PTS: 1

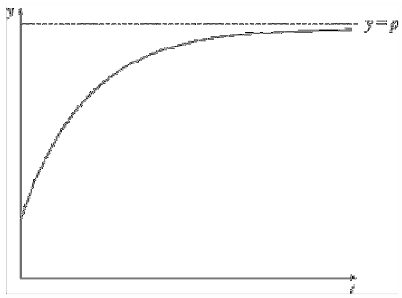
120. ANS:

(a) $y(t) = \dfrac{P}{1 + (P-1)e^{-P \cdot kt}}$

(b) At initial time there were 1000 infected individuals.

(c) $\lim_{t \to \infty} y(t) = P$, which is the total population in this situation.

(d)



PTS: 1

121. ANS:

(a) $x(t) = ce^{\left(k/w\right)\sin(wt)}$

(b) The solutions are periodic functions which model seasonal change in daylight hours.

PTS: 1

122. ANS:

(a) $\dfrac{ds}{dt} = 0.2 - \dfrac{s}{200} \Rightarrow s(t)40 - 35e^{-\left(1/200\right)t}$

(b) $s(20) = 40 - 35e^{-0.1} \approx 8.33$ lb

(c) 40 lb; 20%

PTS: 1

123. ANS:

(a) $\dfrac{dZ}{dt} = -\dfrac{Z}{5} \Rightarrow Z(t) = 4e^{-\left(1/5\right)t}$

(b) $Z(20) = 4e^{-4} \approx 0.073$ lb

(c) $t = 5\ln 4 \approx 6.93$ min

PTS: 1

124. ANS:

(a) $\dfrac{ds}{dt} = -\dfrac{s}{50} \Rightarrow s(t)10e^{-\left(1/50\right)t}$

(b) $s(10) = 10e^{-1/5} \approx 8.19$ kg

PTS: 1

125. ANS:

$\dfrac{dy}{dt} = -\dfrac{8}{500 + 2t}$ with initial condition $y(0) = 10$

PTS: 1

126. ANS:

(a) $\dfrac{ds}{dt} = 0.6 - \dfrac{13s}{1000} \Rightarrow s(t) = \dfrac{600}{13}\left(1 - e^{-3/1000t}\right)$

(b) $s(30) = \dfrac{600}{13}\left(1 - e^{-9/100}\right) \approx 3.97$ kg

(c) $\dfrac{600}{13}$ kg

PTS: 1

127. ANS:

$\dfrac{dc}{dt} = \dfrac{3}{50} - \dfrac{10c}{1000-4t}$, $c(t) = \dfrac{1}{50}\left[500 - 2t - \dfrac{(500-2t)^{5/2}}{500^{3/2}}\right]$, $c(125) \approx 3.23$

PTS: 1

128. ANS:

$\dfrac{dT}{dt} = k(T-72)$, $T(0) = 250$ °F, $T(1) = 200$°F $\Rightarrow T(t) = 72 + 178\left(\dfrac{128}{178}\right)^t$

PTS: 1

129. ANS:

(a) $\dfrac{dT}{dt} = k\left(T_o - T\right)\ \ k > 0$

(b) $\dfrac{dT}{dt} = k\left(70° - T(t)\right)\ \ T(0) = 200$ °F, $T(t) = 70 + 130\left(\dfrac{11}{13}\right)^{t/2}\ \ t \approx 17.56$ minutes

PTS: 1

130. ANS:

(a) $\dfrac{dT}{dt} = k\left(T_o - T\right)\ \ k > 0$

(b) $\dfrac{dT}{dt} = k\left(70° - T(t)\right)\ \ T(0) = 30$ °F, $T(t) = 70 - 40\left(\dfrac{7}{8}\right)^{t/10}\ \ t \approx 21.5$ minutes

PTS: 1

131. ANS:

$1 - 2\sum_{n=0}^{\infty}(-1)^n x^{n+1}$, interval of convergence $|x| < 1$

PTS: 1

132. ANS:
$$\sum_{n=0}^{\infty}(-1)^n\left(\frac{x}{5}\right)^{n+1}$$, interval of convergence $|x| < 5$

PTS: 1

133. ANS:
$$-\frac{1}{2}\sum_{n=0}^{\infty}\frac{x^{n+1}}{n+1}$$, radius of convergence 1

PTS: 1

134. ANS:
$$3\sum_{n=0}^{\infty}\frac{(-9)^n x^{2n+1}}{2n+1}$$, radius of convergence $\frac{1}{3}$

PTS: 1

135. ANS:
$$C+\sum_{n=0}^{\infty}\frac{(-1)^n x^{5n+1}}{5n+1}$$

PTS: 1

136. ANS:
$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^{2n}}{n}$$

PTS: 1

137. ANS:
$$\sum_{n=0}^{\infty}(n+1)x^n$$; radius of convergence 1

PTS: 1

138. ANS:
0.746824

PTS: 1

139. ANS:
Answers will vary.

PTS: 1

140. ANS:
$$\frac{x^2}{(x-1)^2}$$

PTS: 1

141. ANS:
$$\sum_{n=0}^{\infty} 3^n x^{n+2}$$

PTS: 1

142. ANS:
0.100000

PTS: 1

143. ANS:
0.448414

PTS: 1

144. ANS:
$$\frac{3}{2}$$

PTS: 1

145. ANS:
$$\sum_{n=1}^{\infty} n x^{n-1}$$

PTS: 1

146. ANS:
$$\frac{2}{3}\sum_{n=0}^{\infty}(-1)^n\left(\frac{x}{3}\right)^n - 3\sum_{n=0}^{\infty}x^n$$

PTS: 1

147. ANS:
$$\sum_{n=0}^{\infty}\frac{x^{3n+2}}{3n+2}+C$$

PTS: 1

148. ANS:
$$\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)-\frac{\sqrt{2}}{2}\cdot\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2-\frac{\sqrt{2}}{2}\cdot\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\cdots$$

PTS: 1

149. ANS:

$$\ln 2+\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\left(\frac{x-2}{2}\right)^{n}$$

PTS: 1

150. ANS:

$\ln(1+x)=x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}-\frac{1}{4}x^{4}$; $\ln(1.1)=0.09531$, accurate to five decimal places

PTS: 1

151. ANS:

$\sum_{n=1}^{\infty}-\frac{1}{n}x^{n}$; converges for all x such that $-1\le x<1$

PTS: 1

152. ANS:

–150

PTS: 1

153. ANS:

$$\sum_{n=0}^{\infty}\frac{(-1)^{n}x^{2n+1}}{(2n)!}$$

PTS: 1

154. ANS:

Since $\sin t=\sum_{n=0}^{\infty}\frac{(-1)^{n}t^{2n+1}}{(2n+1)!}$, $\frac{\sin t}{t}=\sum_{n=0}^{\infty}\frac{(-1)^{n}t^{2n}}{(2n+1)!}\Rightarrow\int_{0}^{x}\frac{\sin t}{t}\,dt=\sum_{n=0}^{\infty}\int_{0}^{x}\frac{(-1)^{n}t^{2n}}{(2n+1)!}\,dt=\sum_{n=0}^{\infty}\frac{(-1)^{n}x^{2n+1}}{(2n+1)!\,(2n+1)}$

PTS: 1

155. ANS:

Since $\cos t=\sum_{n=0}^{\infty}\frac{(-1)^{n}t^{2n}}{(2n)!}$, $\cos\sqrt{t}=\sum_{n=0}^{\infty}\frac{(-1)^{n}t^{n}}{(2n)!}\Rightarrow\int_{0}^{x}\cos\sqrt{t}\,dt=\sum_{n=0}^{\infty}\int_{0}^{x}\frac{(-1)^{n}t^{n}}{(2n)!}\,dt=\sum_{n=0}^{\infty}\frac{(-1)^{n}x^{n+1}}{(2n)!\,(n+1)}$

PTS: 1

156. ANS:

Since $\frac{1}{1-t^{5}}=\sum_{n=0}^{\infty}(t^{5})^{n}$, $\frac{t}{1-t^{5}}=\sum_{n=0}^{\infty}t^{5n+1}\Rightarrow\int_{0}^{x}\frac{t}{1-t^{5}}\,dt=\sum_{n=0}^{\infty}\int_{0}^{x}t^{5n+1}\,dt=\sum_{n=0}^{\infty}\frac{x^{5n+2}}{(5n+2)}$

PTS: 1

157. ANS:

(a) $\sin t = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!} \Rightarrow \sin(t^2) = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2(2n+1)}}{(2n+1)!}$ and thus

$$\int_0^x \sin(t^2)\,dt = \sum_{n=0}^{\infty} \int_0^x \frac{(-1)^n t^{2(2n+1)}}{(2n+1)!}\,dt = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(2n+1)!\,(4n+3)}$$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n (0.3)^{4n+3}}{(2n+1)!\,(4n+3)}$

PTS: 1

158. ANS:

(a) $\sin t = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!} \Rightarrow \sin(t^2) = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2(2n+1)}}{(2n+1)!}$ and $\frac{\sin(t^2)}{t^2} = \sum_{n=0}^{\infty} \frac{(-1)^n t^{4n}}{(2n+1)!}$, thus

$$\int_0^x \frac{\sin(t^2)}{t^2}\,dt = \sum_{n=0}^{\infty} \int_0^x \frac{(-1)^n t^{4n}}{(2n+1)!}\,dt = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(2n+1)!\,(4n+1)}$$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n (0.3)^{4n+1}}{(2n+1)!\,(4n+1)}$

PTS: 1

159. ANS:

$1 + (\ln 2)x + \frac{(\ln 2)^2 x^2}{2!} + \frac{(\ln 2)^3 x^3}{3!} + \frac{(\ln 2)^4 x^4}{4!} + \frac{(\ln 2)^5 x^5}{5!} + \cdots$; $2^{0.1} \approx 1.0718$

PTS: 1

160. ANS:

$\frac{1}{8}$

PTS: 1

161. ANS:

$\frac{3\sqrt{3}}{2}$

PTS: 1

162. ANS:

$\frac{1}{e} - 1$

PTS: 1

163. ANS:
12

PTS: 1

164. ANS:
$\frac{1}{2}\left(e+\frac{1}{e}\right)$

PTS: 1

165. ANS:
0

PTS: 1

166. ANS:
$\frac{10!}{5!} = 30{,}240$

PTS: 1

167. ANS:
$-\frac{23!}{4^{11}}$

PTS: 1

168. ANS:
$$1+\sum_{n=1}^{\infty}\frac{(-1)^n 3\cdot 4\cdot 5\cdots(n+2)x^n}{n!} \text{ with } R=1$$

PTS: 1

169. ANS:
$$1+\frac{x^2}{3}+\sum_{n=2}^{\infty}\frac{(-1)^{n-1}2\cdot 5\cdot 8\cdots(3n-4)x^{2n}}{3^n n!} \text{ with } R=1$$

PTS: 1

170. ANS:
$$x^2+\sum_{n=1}^{\infty}\frac{1\cdot 3\cdot 5\cdots(2n-1)x^{3n+2}}{2^n n!} \text{ with } R=1$$

PTS: 1

171. ANS:
$$-1+\frac{x}{5}+\sum_{n=2}^{\infty}\frac{4\cdot 9\cdots(5n-6)x^n}{5^n n!} \text{ with } R=1$$

PTS: 1

172. ANS:

$$1+\frac{1}{3}x+\sum_{n=2}^{\infty}\frac{(-1)^{n+1}\cdot 2\cdot 5\cdot 8\cdots(3n-4)x^n}{3^n\cdot n!}, \text{ for } |x|<1$$

PTS: 1

173. ANS:

$$\frac{\sqrt{2}}{2}\left[1+\sum_{n=1}^{\infty}\frac{(-1)^n 1\cdot 3\cdot 5\cdots(2n-1)x^n}{2^{2n}n!}\right] \text{ with } \left|\frac{x}{2}\right|<1 \text{ so } |x|<2 \text{ and } R=2.$$

PTS: 1

174. ANS:

$$8+3x+\sum_{n=2}^{\infty}\frac{(3)(1)(-1)\cdots(5-2n)x^n}{8^{n-1}n!} \text{ with } \left|\frac{x}{4}\right|<1 \text{ so } |x|<4 \text{ and } R=4.$$

PTS: 1

175. ANS:

$$1-\frac{1}{2}x^2$$

PTS: 1

176. ANS:

$$1-x+\frac{3}{2}x^2-\frac{5}{2}x^3$$

PTS: 1

177. ANS:
[0.84, 1.16]

PTS: 1

178. ANS:
[–0.282, 0.274]

PTS: 1

179. ANS:

$$-\frac{1}{e}+\frac{1}{2e}(x+1)^2$$

PTS: 1

180. ANS:

$$T_3(x)=\frac{5}{2}x-5x^2+10x^3$$

PTS: 1

181. ANS:
(a) $T_4(x) = 1 + 3(x-2)^4$
(b) 0
(c) As we can see from the form of $T_4(x) = f(x)$, f has the absolute minimum value $f(2) = 1$.

PTS: 1

182. ANS:
$T_4(x) = 1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4$

PTS: 1

183. ANS:
$T_2(x) = 10 + \frac{1}{20}(x-100) - \frac{1}{8000}(x-100)^2$; $|\text{error}| \le 6 \times 10^{-10}$

PTS: 1

184. ANS:
0.099833

PTS: 1

185. ANS:
$T_4(x) = 2 + \frac{x-4}{4} + \frac{(x-4)^2}{64} + \frac{(x-4)^3}{512} - \frac{5(x-4)^2}{16384} + R_4$; $\sqrt{4.2} \approx 2.049390137$; $R_4 \approx 0.0000000171$

PTS: 1

186. ANS:
$\sqrt{6} \approx T_3(6) = 2 + \frac{2}{4} - \frac{2^2}{64} + \frac{2^3}{512} = 2.453125$; $|\text{error}| \le 0.005$

PTS: 1

187. ANS:
$x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$

PTS: 1

188. ANS:
(a) $1 + \frac{3}{2}(x+3) + \frac{3}{8}(x+3)^2 - \frac{1}{16}(x+3)^3$
(b) $8 + 3x + \frac{3}{16}x^2$
(c) Error $\le \frac{Mx^3}{3!}$ where $M = \max\left|f^{(3)}(x)\right| = \frac{3}{64}$. So error $\le \frac{3}{64 \cdot 6}x^3 = \frac{x^3}{128}$

PTS: 1

189. ANS:
(a) x
(b) $x+\frac{x^3}{3}$
(c) $x+\frac{x^3}{3}+\frac{2}{15}x^5$

PTS: 1

190. ANS:
$2+x^2-5x^3$. The remainder is 0

PTS: 1

191. ANS:
(a) $x+\frac{x^3}{6}$
(b) About 0.201333
(c) A calculator gives 0.2013579.

PTS: 1

192. ANS:
(a) About 0.382
(b) $\cos 1+\sin 1-1$
(c) $\cos 1+\sin 1-1\approx 0.38177$

PTS: 1

193. ANS:
About 0.7635

PTS: 1

194. ANS:
$T_3(x)=(x-1)-\frac{1}{2!}(x-1)^2+\frac{2}{3!}(x-1)^3$; $\ln(1.2)\approx 0.18266$

PTS: 1

195. ANS:
$\frac{1}{3}x^3$

PTS: 1

196. ANS:
$1+\frac{x^3}{2}$

PTS: 1

197. ANS:

$$-\frac{1}{e}+\frac{e}{2}\left(x-\frac{1}{e}\right)^2$$

PTS: 1