Practice Final Exam, problems 13–16

11. Which value is closest to $\int_0^{1/2} \cos(x^2)\,dx$?

    (a) 0.500

    (b) 0.497

    (c) 0.495

    (d) 0.480

    (e) 0.478

    (f) 0.400

12. Suppose $y(t)$ satisfies

$$y' = \frac{1+y^4}{t}$$

    with initial condition $y(0) = b$. Which of the following is a true statement about the behavior of the solution for $t > 0$?

    (a) No matter what the value of b, there is a vertical asymptote.

    (b) No matter what the value of b, there is a horizontal asymptote.

    (c) No matter what the value of b, the value of y goes to infinity as $t \to \infty$.

    (d) more than one of the above behaviors is possible, depending on the value of b.

13. Find an analytic expression for the function $f(x)$ defined by

$$f(x) = \sum_{n=1}^{\infty} n x^{n-1} .$$

14. Compute the first three terms of the Maclaurin series for $\sqrt[3]{1+x}$.

15. Two players take turns rolling a 6-sided die. The winner is the first to roll a 5. What is the probability that the first player wins? [You may assume that the probability of a conjunction of events is the product of the probabilities, for example, the probability that the first player rolls a 6 and then the second player rolls a 4 is 1/36.]

16. Find the terms of degree 0, 1 and 2 of the power series solution to the differential equation

$$\frac{dy}{dx} = x^2y + y^2x$$

with initial condition $y(0) = 1$.

ANSWERS

11 (b)

12 (a)

13 $1/(1-x)^2$

14 $1 + (1/3)x - (1/9)x^2$

15 $6/11$

16 $1 + (1/2)x^2$