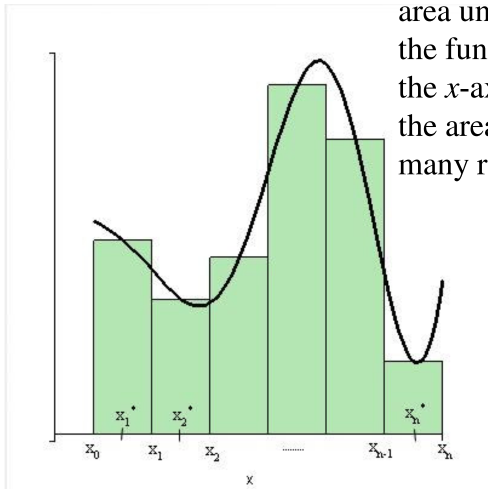
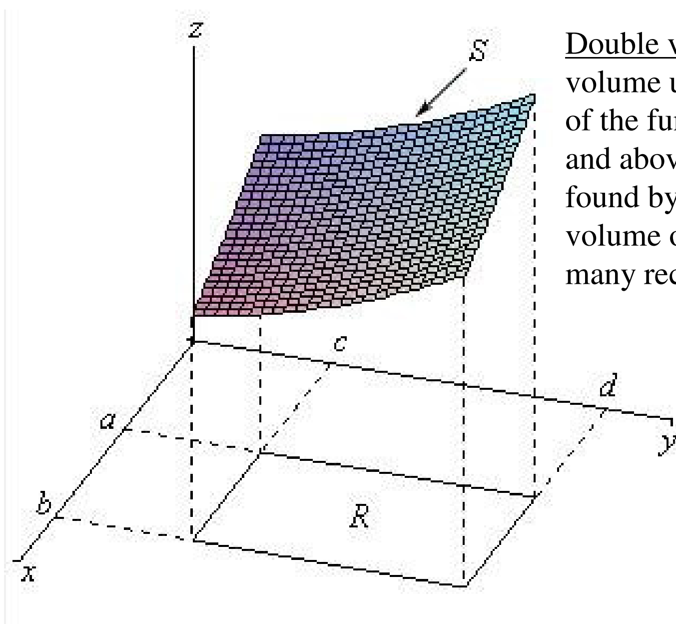
16.1 Double Integrals



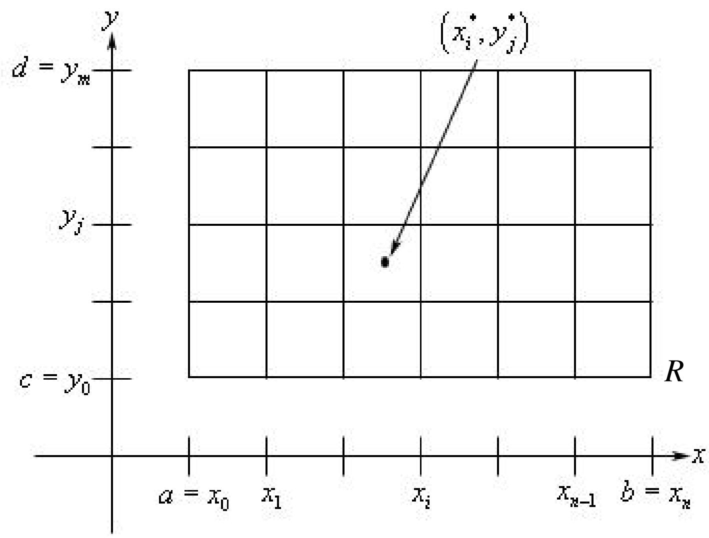
<u>Single variable integral</u> : area under the graph of the function and above the *x*-axis found by using the area of infinitely many rectangles.

$A \approx \sum_{i=1}^{n} f\left(x_i^*\right) \Delta x$ $A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(x_i^*\right) \Delta x$ $A = \int_a^b f(x)\, dx$

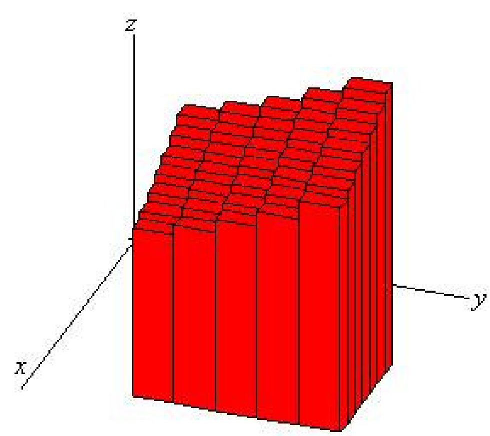


<u>Double variable integral</u> : volume under the graph of the function (surface) and above the *xy*-plane found by using the volume of infinitely many rectangular prisms.

16.1



16.1

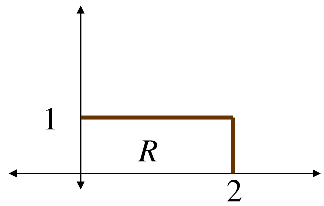


$$V \approx \sum_{i=1}^{n} \sum_{j=1}^{m} f\left(x_i^*, y_j^*\right) \Delta A \qquad V = \lim_{n,m \to \infty} \sum_{i=1}^{n} \sum_{j=1}^{m} f\left(x_i^*, y_j^*\right) \Delta A$$

$$V = \int_c^d \int_a^b f(x, y)\, dxdy = \int_a^b \int_c^d f(x, y)\, dydx$$

16.1

When the region R is rectangular, the integration is simplified.



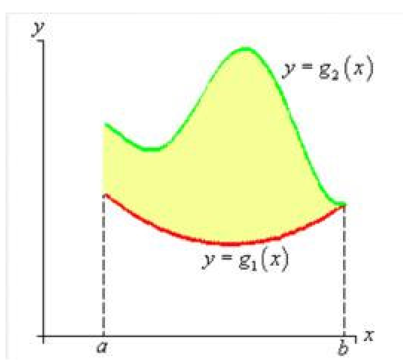
$$\int_0^1\left(\int_0^2 4-x-y\,dx\right)dy = \int_0^1\left(4x-\frac{x^2}{2}-xy\Big|_0^2\right)dy=\int_0^1\left[(8-2-2y)-0\right]dy$$

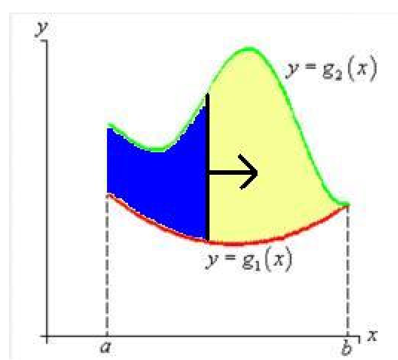
$$=\int_0^1 6-2y\,dy = 6y-y^2\Big|_0^1 = 5$$

This is called an iterated integral.

16.2

Double Integrals over general regions



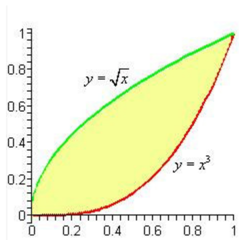


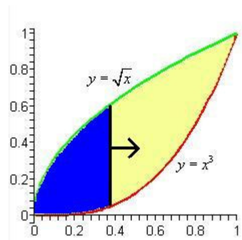
dydx

$$\iint_R f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dydx$$

16.3

$$\iint_R 4xy - y^3 \, dA \quad R = \text{the region between } y = \sqrt{x} \text{ and } y = x^3$$





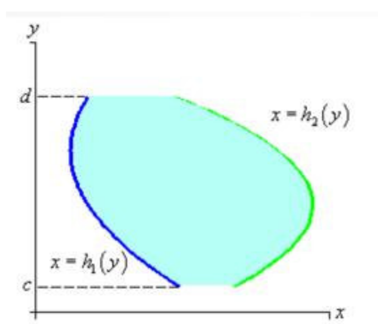
$$\iint_R 4xy - y^3 \, dydx$$

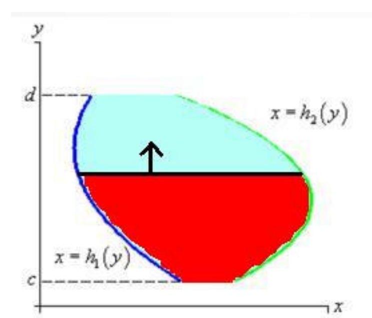
$$\int_0^1 \int_{x^3}^{\sqrt{x}} 4xy - y^3 \, dydx = \int_0^1 \left[2xy^2 - \frac{y^4}{4} \right]_{x^3}^{\sqrt{x}} dx$$

$$= \int_0^1 \underbrace{\left(2x^2 - \frac{x^2}{4}\right)}_{\frac{7x^2}{4}} - \left(2x^7 - \frac{x^{12}}{4}\right) dx = \int_0^1 \left(\frac{7x^2}{4} - 2x^7 + \frac{x^{12}}{4}\right) dx = \left(\frac{7x^3}{12} - \frac{x^8}{4} + \frac{x^{13}}{52}\right)\Bigg|_0^1$$

$$= \frac{7}{12} - \frac{1}{4} + \frac{1}{52} = \frac{1}{3} + \frac{1}{52} = \boxed{\frac{55}{156}}$$

16.3



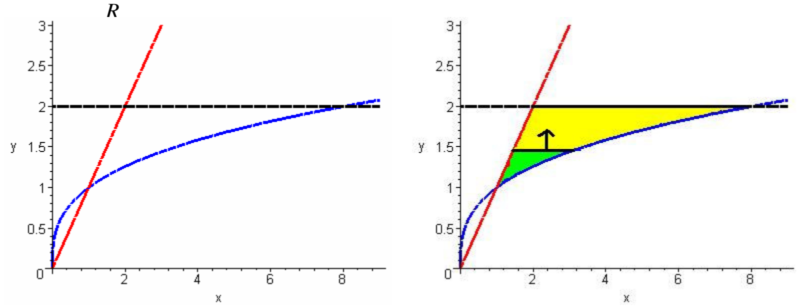


$$dxdy$$

$$\iint_R f(x, y) \, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) \, dxdy$$

16.3

16.3 $\iint\limits_R e^{x/y}\,dA \quad R = \text{the region } 1 \le y \le 2,\ y^3 \le x \le y$



$$\iint\limits_R e^{x/y}\,dxdy$$
$$= \int_1^2 \int_y^{y^3} e^{x/y}\,dxdy = \int_1^2 \left[ye^{x/y} \right]_y^{y^3} dy = \int_1^2 \left(ye^{y^2} - ye^1 \right) dy$$
$$= \left[\frac{1}{2}e^{y^2} - \frac{y^2}{2}e^1 \right]_1^2 = \left(\frac{1}{2}e^4 - 2e \right) - \left(\frac{1}{2}e - \frac{1}{2}e \right) = \boxed{\frac{1}{2}e^4 - 2e}$$

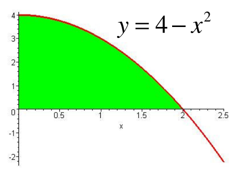
Area of a 2-dimensional region using double integration

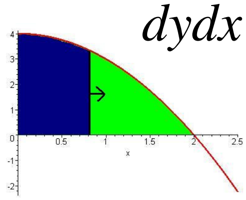
Use $f(x, y) = 1 \qquad \iint\limits_R dA$

$$\int_a^b \int_{g_1(x)}^{g_2(x)} dydx = \int_a^b \left[g_2(x) - g_1(x) \right] dx$$

$$\int_c^d \int_{h_1(y)}^{h_2(y)} dxdy = \int_c^d \left[h_2(y) - h_1(y) \right] dy$$

16.3





dydx

$$\int_a^b \int_{g_1(x)}^{g_2(x)} dydx = \int_0^2 \int_0^{4-x^2} dydx = \int_0^2 \left[4 - x^2\right] dx$$

$$= \left[4x - \frac{x^3}{3}\right]_0^2 = 8 - \frac{8}{3} = \boxed{\frac{16}{3}}$$

16.3

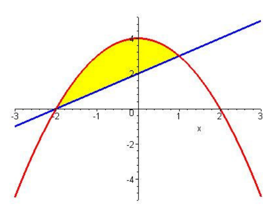
Order of Integration dictated by:

a) the integrand

$$\int_0^2 \int_x^2 e^{-y^2} dydx$$

b) the region

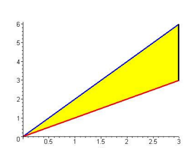
$$\iint_R dA \quad R: x + 2 \le y \le 4 - x^2$$



c) Both the integrand and the region

$$\iint_R \frac{y}{x^2 + y^2} \quad R:\ \text{triangle bounded}$$

by $y = x, y = 2x, x = 3$



16.3