

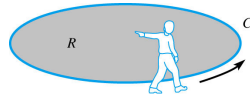
9.12 Green's Theorem

9.12
Green's
Theorem
-Rimmer

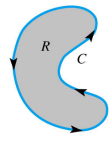
Closed Curve Line Integral

$$\oint Pdx + Qdy$$

Closed Curve Orientation:



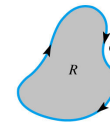
(a) Positive direction



(b) Positive direction

Counter-clockwise

$$\oint_C Pdx + Qdy$$



(c) Negative direction

Clockwise

$$\oint Pdx + Qdy = -\oint_C Pdx + Qdy$$

Green's Theorem (in the plane = 2 dim.)

9.12
Green's
Theorem
-Rimmer

Suppose that C is a piecewise smooth simple closed curve bounding a region R .

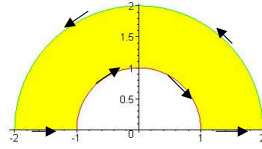
If $P, Q, P_y,$ and Q_x are continuous on R , then

$$\oint_C Pdx + Qdy = \iint_R (Q_x - P_y) dA$$

$$\oint_C y^2 dx + 3xy dy$$

$$P = y^2, Q = 3xy$$

$$Q_x = 3y, P_y = 2y$$



$$\oint_C y^2 dx + 3xy dy \stackrel{GThm}{=} \iint_R y dA = \int_0^{\pi/2} \int_1^2 r \sin \theta r dr d\theta$$

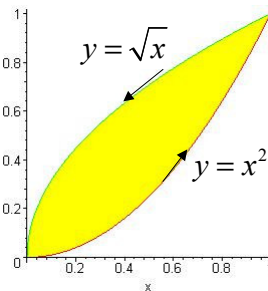
$$= \int_0^{\pi/2} \sin \theta d\theta \int_1^2 r^2 dr$$

$$= \frac{7}{3} \int_0^{\pi/2} \sin \theta d\theta = \boxed{\frac{14}{3}}$$

$$\oint_C (y + e^{\sqrt{x}}) dx + (2x + \cos y^2) dy$$

$$P = y + e^{\sqrt{x}}, Q = 2x + \cos y^2$$

$$Q_x = 2, P_y = 1$$



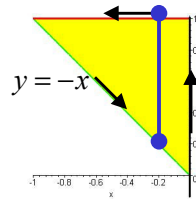
$$\oint_C (y + e^{\sqrt{x}}) dx + (2x + \cos y^2) dy \stackrel{GThm}{=} \iint_R dA$$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} dy dx = \int_0^1 (\sqrt{x} - x^2) dx$$

$$= \left[\frac{2}{3} x^{3/2} - \frac{1}{3} x^3 \right]_0^1 = \boxed{\frac{1}{3}}$$

$$\oint_C (e^{x^2}) dx + (2 \arctan x) dy$$

C : triangle with vertices
 $(0,0)$, $(0,1)$, and $(-1,1)$



$$P = e^{x^2}, Q = 2 \arctan x \Rightarrow Q_x = \frac{2}{1+x^2}, P_y = 0$$

$$\oint_C (e^{x^2}) dx + (2 \arctan x) dy \stackrel{GThm}{=} \iint_R \frac{2}{1+x^2} dA = \int_{-1}^0 \int_{-x}^1 \frac{2}{1+x^2} dy dx$$

$$= \int_{-1}^0 \left[\frac{2}{1+x^2} y \right]_{-x}^1 dx = \int_{-1}^0 \left(\frac{2}{1+x^2} + \frac{2x}{1+x^2} \right) dx$$

$$= \left[2 \arctan x + \ln(1+x^2) \right]_{-1}^0$$

$$= 0 - \left[2 \arctan(-1) + \ln(2) \right] = - \left[2 \left(-\frac{\pi}{4} \right) + \ln(2) \right] = \boxed{\frac{\pi}{2} - \ln(2)}$$