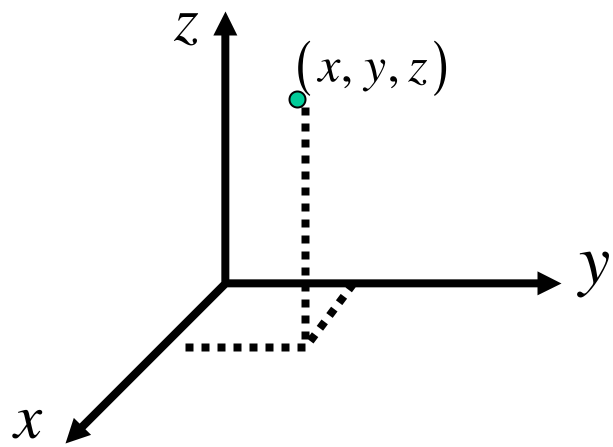
$$\iiint_D F(x,y,z)\,dV \qquad dV = dz\,dy\,dx \text{ in any order}$$

$$F = 1 \Rightarrow \text{Volume} = \iiint_D dV$$



Rectangular Coordinates

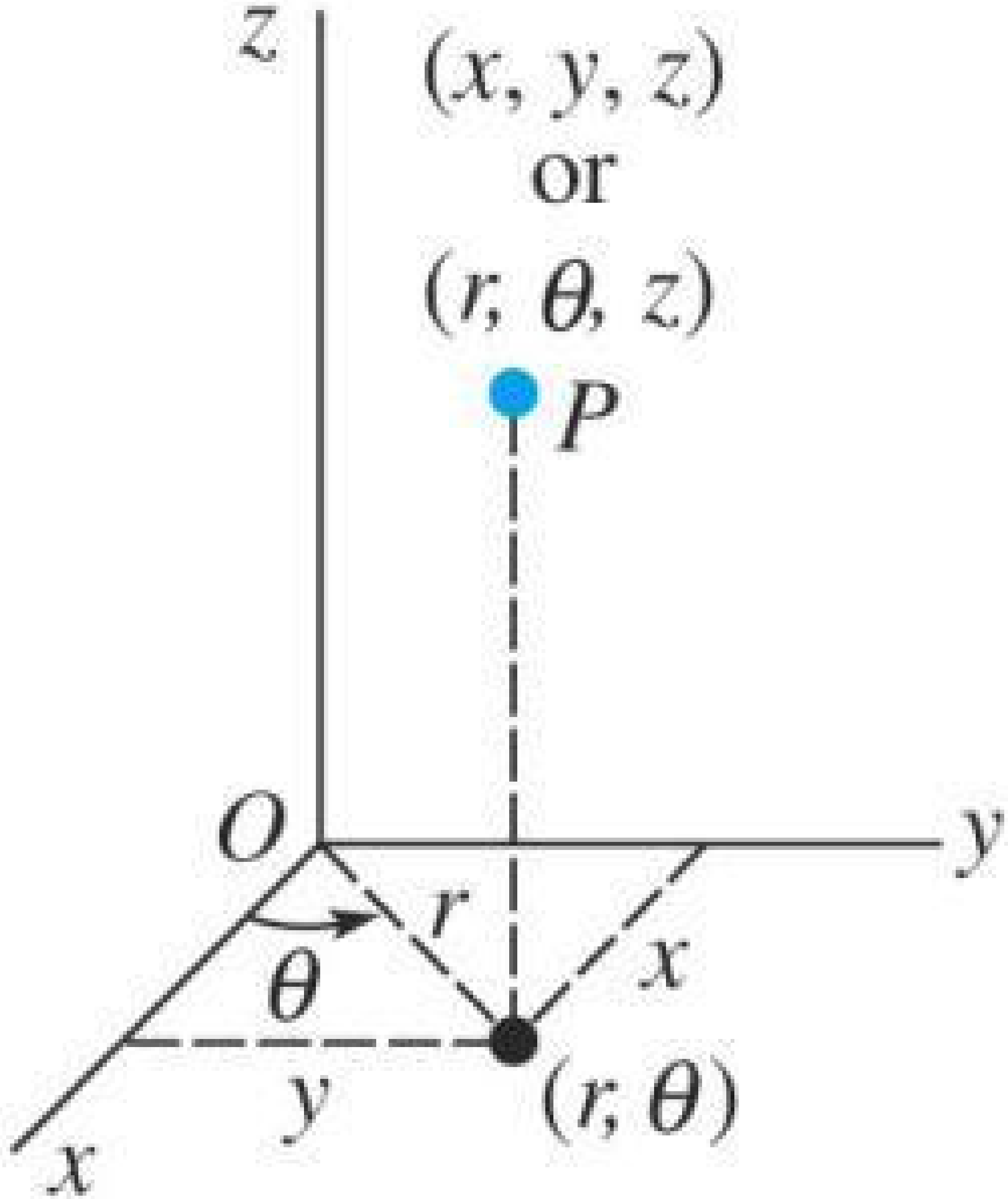
(x, y, z)

$$\int_0^{\frac{\pi}{2}}\int_0^{y^2}\int_0^{y}\cos\left(\frac{x}{y}\right)dz\,dx\,dy = \int_0^{\frac{\pi}{2}}\int_0^{y^2}\left[z\cos\left(\frac{x}{y}\right)\right]_0^{y} dx\,dy = \int_0^{\frac{\pi}{2}}\int_0^{y^2} y\cos\left(\frac{x}{y}\right)dx\,dy$$

$$= \int_0^{\frac{\pi}{2}}\left[y\,\tfrac{1}{1/y}\sin\left(\frac{x}{y}\right)\right]_{x=0}^{x=y^2} dy = \int_0^{\frac{\pi}{2}} y^2\sin(y)\,dy$$

$\underline{D}$		$\underline{I}$
y^2	$+$	$\sin(y)$
$2y$	$-$	$-\cos(y)$
2	$+$	$-\sin(y)$
0		$\cos(y)$

$$= \left[-y^2\cos(y) + 2y\sin(y) + 2\cos(y)\right]_0^{\pi/2} = \boxed{\pi - 2}$$



Cylindrical Coordinates

$$(r, \theta, z)$$

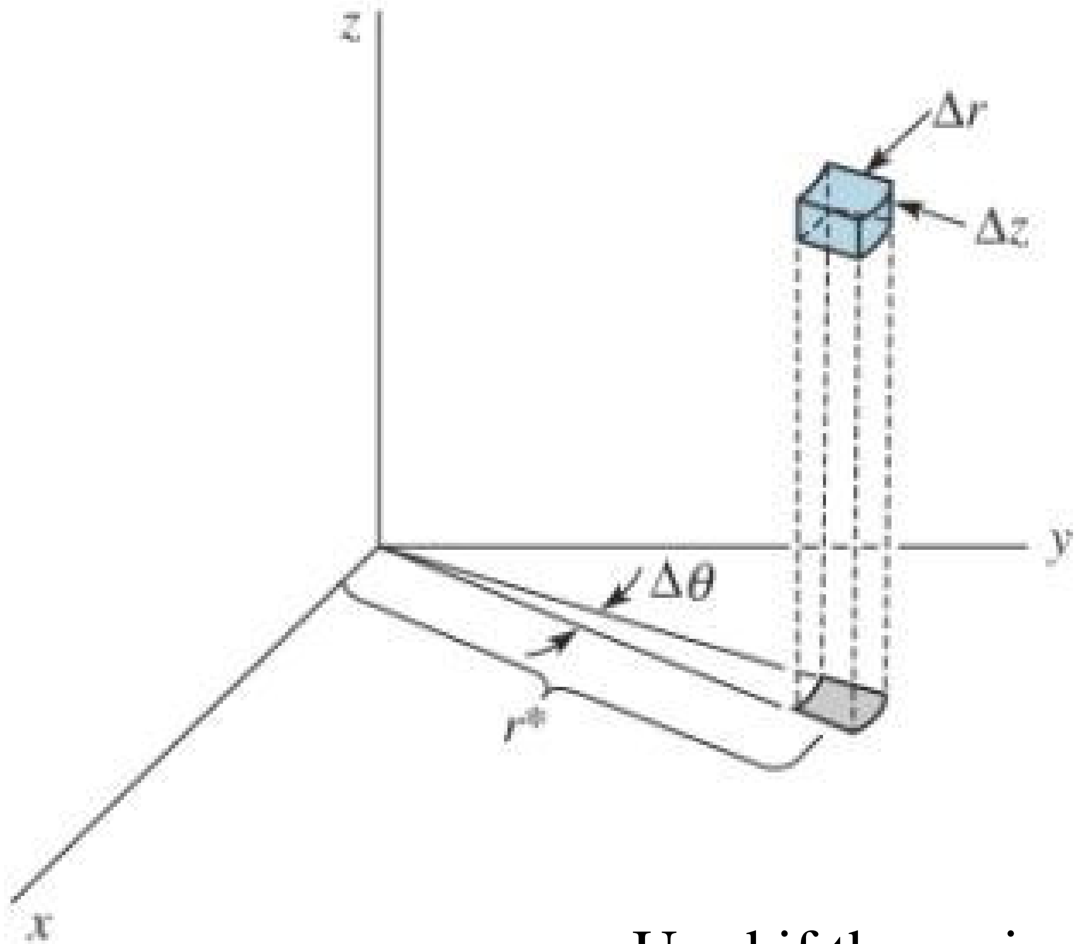
$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$z = z$$

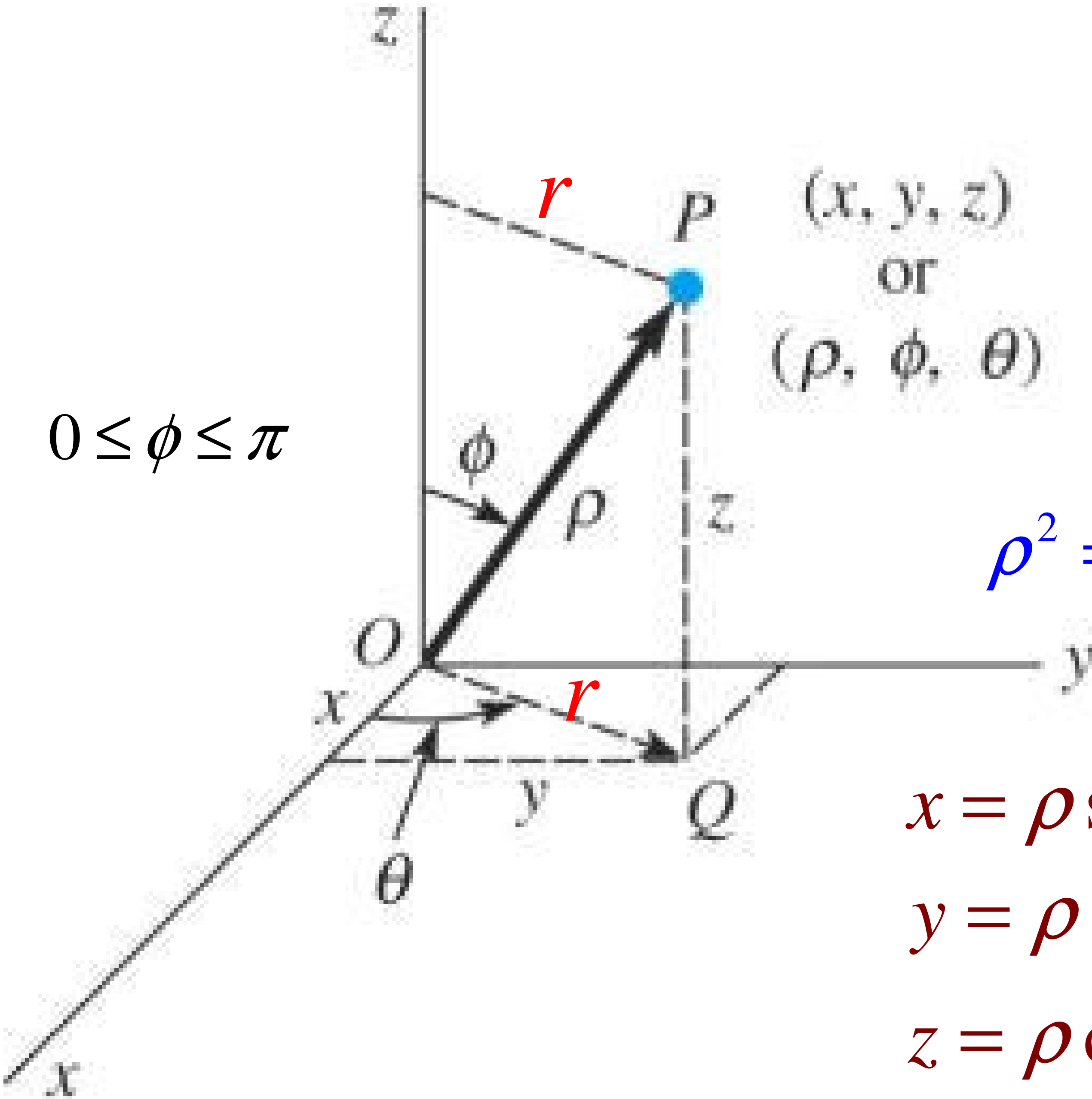


$$\iiint_D F(r,\theta,z)\,dV$$

$$dV = r\,dz\,dr\,d\theta$$

Used if the region in the xy – plane is circular or a portion of a circle

Spherical Coordinates



$$0 \le \phi \le \pi$$

$$(\rho, \phi, \theta)$$

$$\rho^2 = r^2 + z^2$$

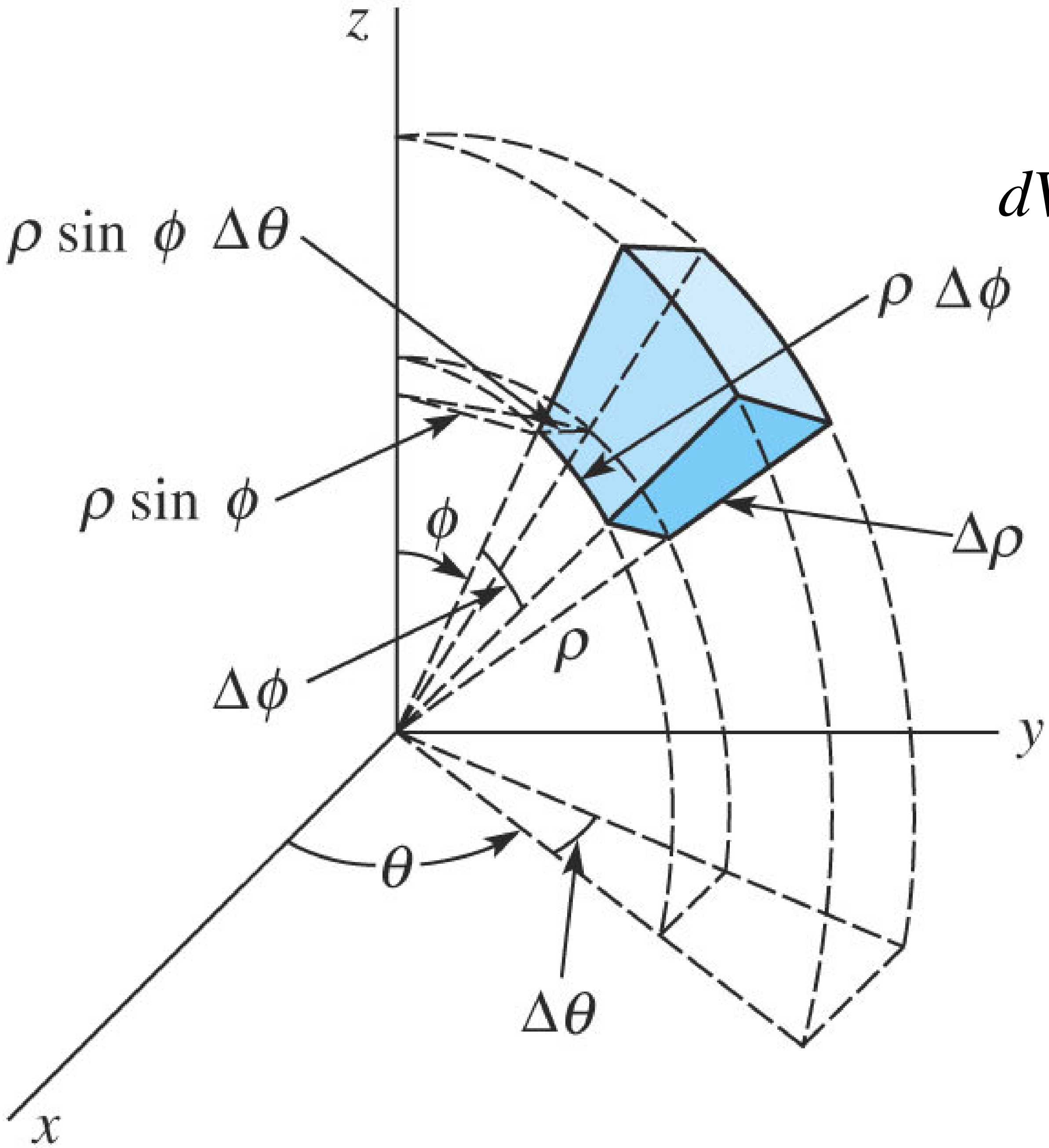
$$\rho^2 = x^2 + y^2 + z^2$$

$$r = \rho \sin \phi$$

$$x = \rho \sin \phi \cos \theta$$

$$y = \rho \sin \phi \sin \theta$$

$$z = \rho \cos \phi$$



$$\iiint_D F(\rho,\phi,\theta)\,dV$$

$$dV = \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$$