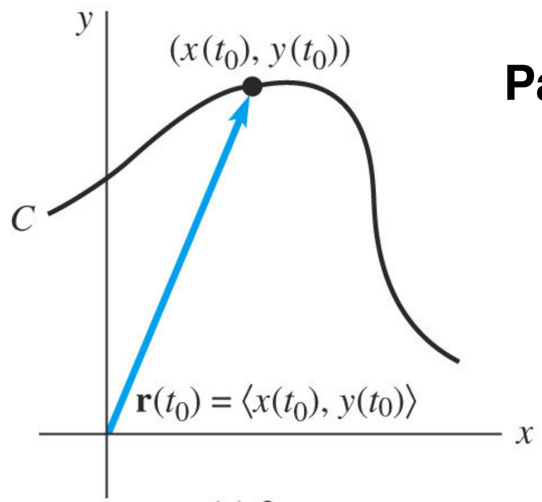
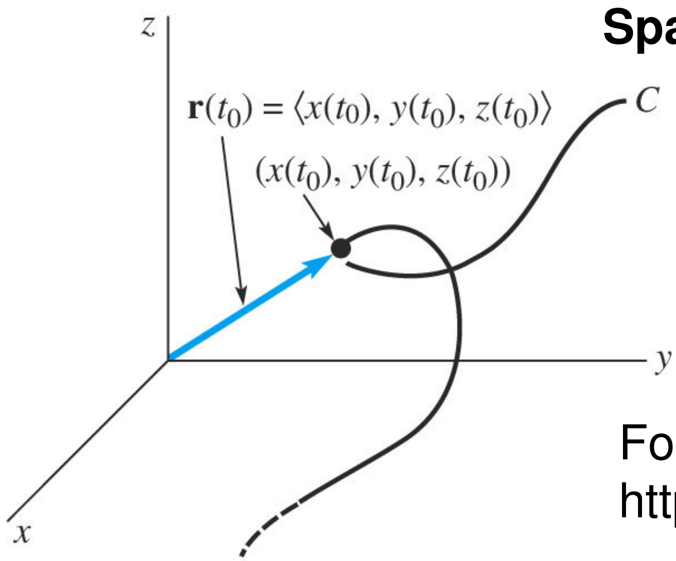
9.1 Vector valued functions or vector functions



(a) 2-space

Parametric Curve



(b) 3-space

Space Curve

For interesting animations of space curves go to:
http://math.bu.edu/people/paul/225/fall05/class6.html

Graph the curve traced by the given vector function.

$r(t) = \langle 4, 2\cos(t), 2\sin(t) \rangle$

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> spacecurve([4, 2*cos(t), 3*sin(t)], t=0..10, thickness=3, color=black);
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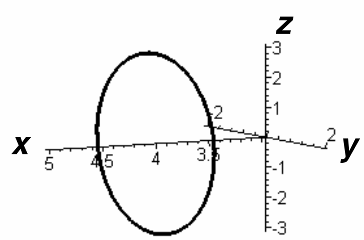

Graph the curve traced by the given vector function.

$r(t) = \langle t\cos(t), t\sin(t), t^2 \rangle$

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> spacecurve([t*cos(t), t*sin(t), t^2], t=0..10, thickness=3, color=black);
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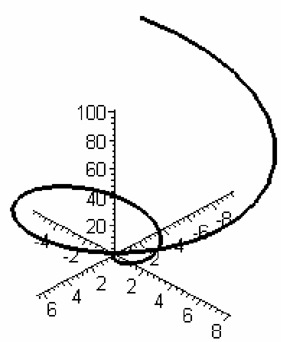


9.1

Find parametric equations of the tangent line to the given curve at the indicated value of t.

$$r(t)=\left\langle t^3-t,\frac{6t}{t+1},(2t+1)^2\right\rangle \quad t=1$$

$$r'(t)=\left\langle 3t^2-1,\frac{6(t+1)-6t}{(t+1)^2},2(2t+1)\cdot 2\right\rangle$$

$$r'(t)=\left\langle 3t^2-1,\frac{6}{(t+1)^2},4(2t+1)\right\rangle$$

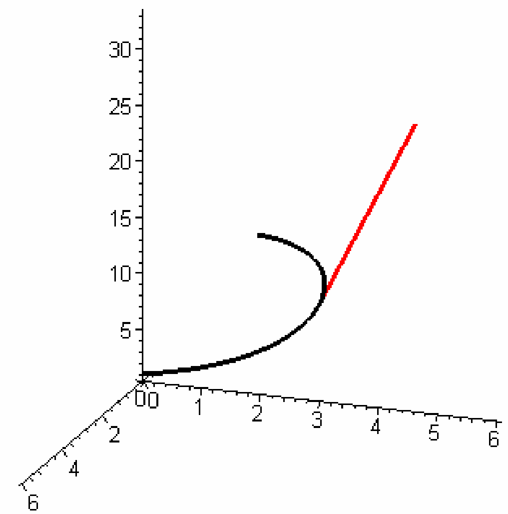
$$r'(1)=\left\langle 3-1,\frac{6}{(2)^2},4(3)\right\rangle=\left\langle 2,\frac{3}{2},12\right\rangle$$

This is a direction vector for the tangent line, we need a pt. on the line.

$r(1)$ will give the point of tangency.

$r(1)=\langle 0,3,9\rangle$ so the point is $(0,3,9)$

		pt.		*dir*
x	=	0	+	$2t$
y	=	3	+	$\frac{3}{2}t$
z	=	9	+	$12t$



9.1

Evaluate the given integral.

$$\int_0^4 \left(\sqrt{2t+1}\ \mathbf{i}\ -\sqrt{t}\ \mathbf{j}\ +\sin(\pi t)\ \mathbf{k}\right)dt$$

$$\int_0^4 \sqrt{2t+1}\ dt\ \mathbf{i}\ -\int_0^4 \sqrt{t}\ dt\ \mathbf{j}\ +\ \int_0^4 \sin(\pi t)\ dt\ \mathbf{k}$$

$$=\left\langle \frac{1}{3}(2t+1)^{3/2}\Big|_0^4, -\frac{2}{3}t^{3/2}\Big|_0^4, -\frac{1}{\pi}\cos(\pi t)\Big|_0^4 \right\rangle$$

$$=\left\langle \frac{1}{3}\left(9^{3/2}-1\right), -\frac{2}{3}\left(4^{3/2}-0\right), -\frac{1}{\pi}\left(\cos(4\pi)-\cos 0\right) \right\rangle$$

$$=\boxed{\left\langle \frac{26}{3}, -\frac{16}{3}, 0 \right\rangle}$$

Find the length of the curve traced by the given vector function on the indicated interval.

$$r(t)=\left\langle 2t, \ln t, t^2 \right\rangle \quad 1 \le t \le 4$$

Arc Length $\rightarrow s=\int_a^b \left\| r'(t) \right\| dt$

$$r'(t)=\left\langle 2, \frac{1}{t}, 2t \right\rangle$$

$$\left\| r'(t) \right\| = \sqrt{4+\frac{1}{t^2}+4t^2}$$

$$\left\| r'(t) \right\| = \sqrt{\frac{4t^2+1+4t^4}{t^2}} = \sqrt{\frac{4t^4+4t^2+1}{t^2}}$$

$$\left\| r'(t) \right\| = \sqrt{\frac{\left(2t^2+1\right)^2}{t^2}}$$

$$\left\| r'(t) \right\| = \frac{2t^2+1}{t} \Rightarrow \left\| r'(t) \right\| = 2t+\frac{1}{t}$$

Arc Length $\rightarrow s=\int_1^4 \left\| r'(t) \right\| dt = \int_1^4 \left(2t+\frac{1}{t}\right)dt$

$$s=\left(t^2+\ln t\right)\Big|_1^4 = 16+\ln 4-1$$

$$\boxed{s=15+\ln 4}$$

9.1