Counting Combinations

Order doesn't matter

$n =$ the total number of objects you are choosing from

$r =$ the number of objects you are choosing

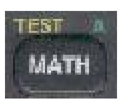
$C_{n,r} =$ total number of ways to choose r different objects out of a total of n when order doesn't matter.

$$C_{n,r} = \frac{n!}{r!(n-r)!}$$

Example: Total number of 5 card hands that can be dealt from a standard 52 card deck

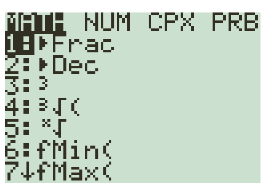
$$C_{52,5} = \frac{52!}{5!(52-5)!} = \frac{52!}{5!\cdot 47!} = \frac{52\cdot 51\cdot \overset{10}{\cancel{50}}\cdot 49\cdot \overset{2}{\cancel{48}}\cdot \cancel{47\cdot 46\cdot\ldots\cdot 2\cdot 1}}{\cancel{5}\cdot \cancel{4\cdot 3\cdot 2\cdot 1}\cdot \cancel{47\cdot 46\cdot 45\cdot\ldots\cdot 2\cdot 1}} = 2{,}598{,}960$$

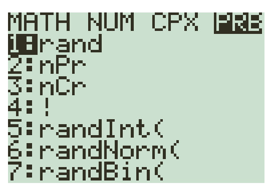
On TI-83:
Type 52 first

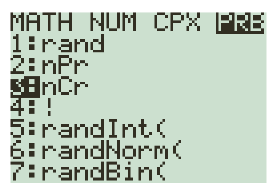


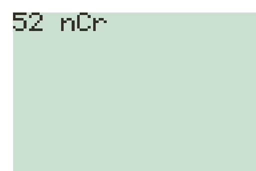
To PRB

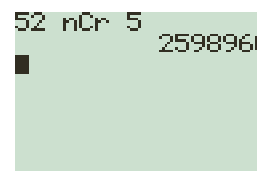
Now type 5











52 Card deck

26 **Black** Cards — 26 **Red** Cards

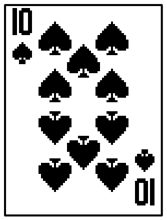
13 Spades — 13 Clubs — 13 Hearts — 13 Diamonds

These are called **suits**

In each suit, you have the following 13 cards:

A	K	Q	J	10	9	8	7	6	5	4	3	2
ace	king	queen	jack									

This is called **rank** of the card



rank: 10

suit: spades

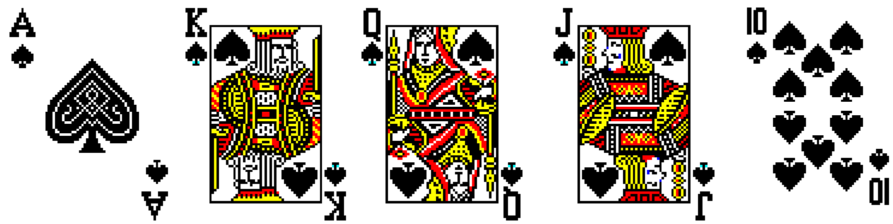
card: 10 of spades

How many ways are there to get 5 cards in rank order ?

$A\ K\ Q\ J\ 10$	9 8 7 6 5
$K\ Q\ J\ 10\ 9$	8 7 6 5 4
$Q\ J\ 10\ 9\ 8$	7 6 5 4 3
$J\ 10\ 9\ 8\ 7$	6 5 4 3 2
10 9 8 7 6	$5\ 4\ 3\ 2\ A$

This is called a straight. (Here we are not considering suits)

Royal Flush - Ace, King, Queen, Jack, 10 all of the same suit



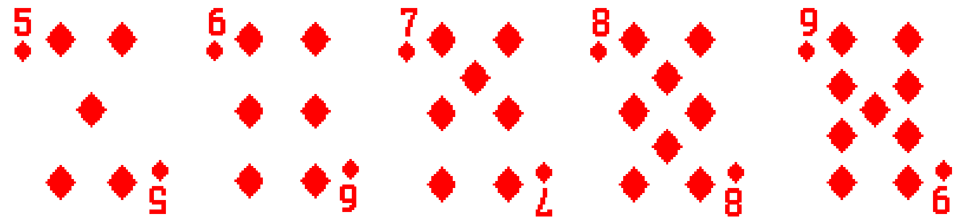
One Royal Flush for each suit

$$P(Royal\ \ Flush)=\frac{\overbrace{C_{4,1}}^{pick\ \text{suit}}}{C_{52,5}}$$

$$P(Royal\ \ Flush)=\frac{4}{2{,}598{,}960}\approx 0.0000015391$$

$$Odds(Royal\ \ Flush)\approx 1\text{ in }649{,}740$$

Straight Flush - Five cards in rank order of the same suit

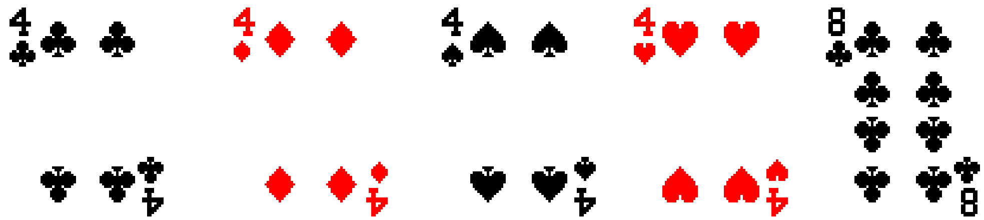


$$P\left(Straight\ \ Flush\right) = \frac{\overbrace{C_{4,1}}^{pick\ \text{suit}} \cdot \overbrace{C_{10,1}}^{pick\ \text{straight}} - \overbrace{4}^{royal}}{C_{52,5}}$$

$$P\left(Straight\ \ Flush\right) = \frac{36}{2{,}598{,}960} \approx 0.0000138517$$

$$Odds\left(Straight\ \ Flush\right) \approx 1 \text{ in } 72{,}193$$

4 of a kind - Four cards of one rank and one other card

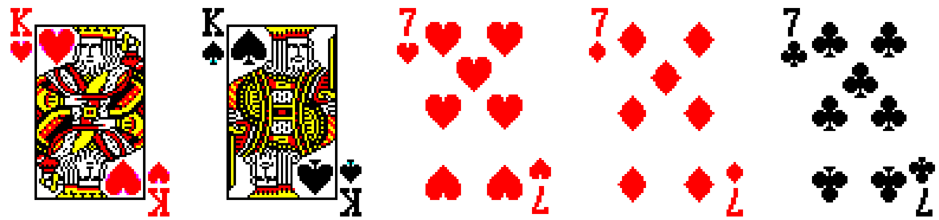


$$P(4\ of\ \ a\ kind) = \frac{\overbrace{C_{13,1}}^{pick\ \text{the rank}} \cdot \overbrace{C_{4,4}}^{pick\ \text{all suits}} \cdot \overbrace{C_{48,1}}^{pick\ \text{other card}}}{C_{52,5}}$$

$$P(4\ of\ \ a\ kind) = \frac{624}{2,598,960} \approx 0.0002400960$$

$$Odds(4\ of\ \ a\ kind) \approx 1 \text{ in } 4,165$$

Full House - Three cards of one rank and two cards of another rank

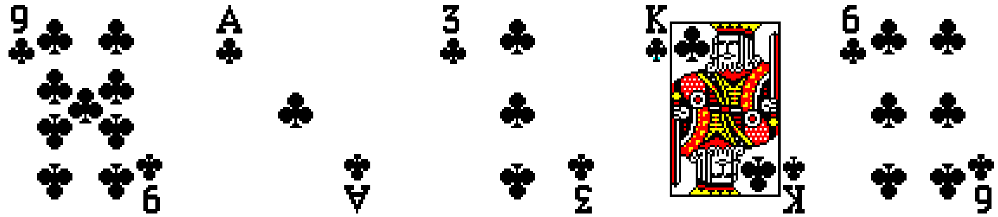


$$P(Full\ House) = \frac{\overbrace{C_{13,1}}^{pick\text{ the rank}} \cdot \overbrace{C_{4,3}}^{pick\text{ the suits}} \cdot \overbrace{C_{12,1}}^{pick\text{ other rank}} \cdot \overbrace{C_{4,2}}^{pick\text{ other suits}}}{C_{52,5}}$$

$$P(Full\ House) = \frac{3,744}{2,598,960} \approx 0.0014405762$$

$$Odds(Full\ House) \approx 1 \text{ in } 694$$

Flush - Five cards of the same suit

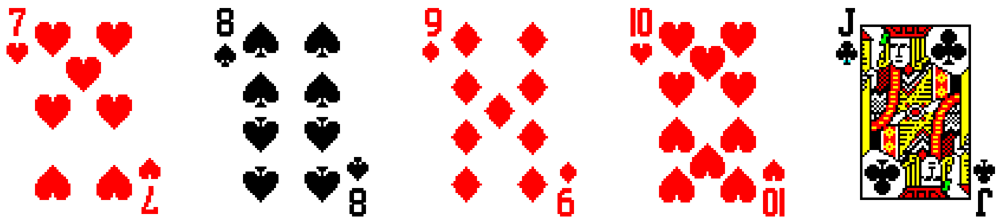


$$P(Flush) = \frac{\overbrace{C_{4,1}}^{\textit{pick}\ \text{suit}} \cdot \overbrace{C_{13,5}}^{\textit{pick}\ 5\ \text{ranks}} - \overbrace{4}^{\textit{royal}} - \overbrace{36}^{\textit{straight flushes}}}{C_{52,5}}$$

$$P(Flush) = \frac{5,108}{2,598,960} \approx 0.0019654015$$

$$Odds(Flush) \approx 1 \text{ in } 509$$

Straight - Five consecutive cards each of any suit

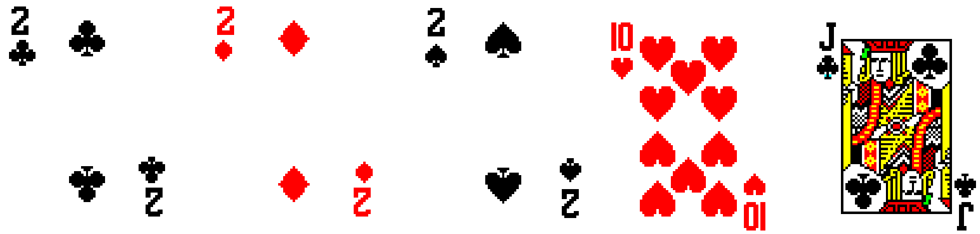


$$P(Straight) = \frac{\overbrace{C_{10,1}}^{pick\ \text{straight}} \cdot \overbrace{C_{4,1} \cdot C_{4,1} \cdot C_{4,1} \cdot C_{4,1} \cdot C_{4,1}}^{pick\ \text{each suit}} - \overbrace{4}^{royal} - \overbrace{36}^{straight\ flushes}}{C_{52,5}}$$

$$P(Straight) = \frac{10,200}{2,598,960} \approx 0.0039246468$$

$$Odds(Straight) \approx 1 \text{ in } 255$$

3 of a Kind – Three cards of one rank and two cards of different rank

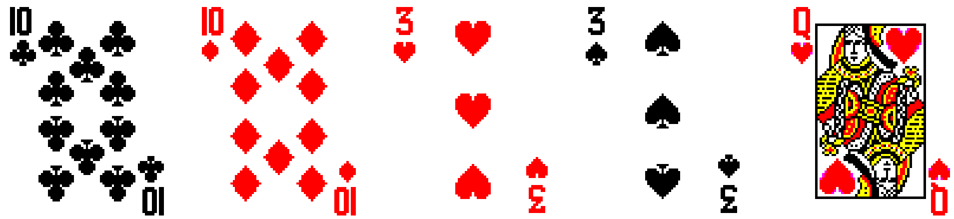


$$P(3 \text{ of a kind}) = \frac{\overbrace{C_{13,1}}^{pick\ \text{the rank}} \cdot \overbrace{C_{4,3}}^{pick\ \text{the suits}} \cdot \overbrace{C_{12,2}}^{pick\ \text{other's rank}} \cdot \overbrace{C_{4,1}}^{pick\ \text{a suit}} \cdot \overbrace{C_{4,1}}^{pick\ \text{a suit}}}{C_{52,5}}$$

$$P(3 \text{ of a kind}) = \frac{54,912}{2,598,960} \approx 0.0211284514$$

$$Odds(3 \text{ of a kind}) \approx 1 \text{ in } 47$$

2 pair - 2 cards of one rank, 2 cards of another rank and another card of a third rank

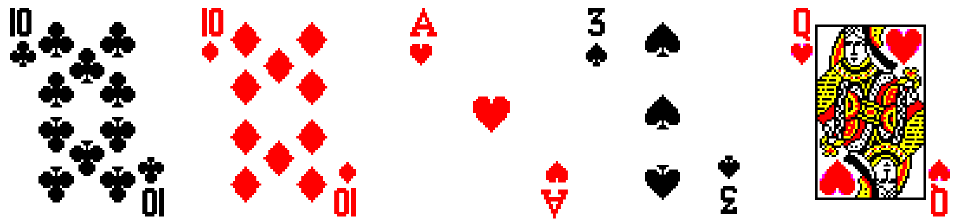


$$P(2\,pair) = \frac{\overbrace{C_{13,2}}^{\textit{pick}\text{ the rank of the pairs}} \cdot \overbrace{C_{4,2}}^{\textit{pick}\text{ the suits}} \cdot \overbrace{C_{4,2}}^{\textit{pick}\text{ the suits}} \cdot \overbrace{C_{44,1}}^{\textit{pick}\text{ other card}}}{C_{52,5}}$$

$$P(2\,pair) = \frac{123{,}552}{2{,}598{,}960} \approx 0.0475390156$$

$$Odds(2\,pair) \approx 1 \text{ in } 21$$

One pair - 2 cards of one rank, 3 other cards of different rank

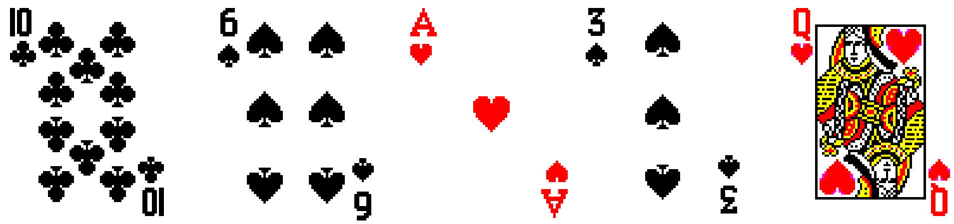


$$P(One\ pair) = \frac{\overbrace{C_{13,1}}^{\textit{pick}\text{ the rank of the pair}} \cdot \overbrace{C_{4,2}}^{\textit{pick}\text{ the suits}} \cdot \overbrace{C_{12,3}}^{\textit{pick}\text{ the other ranks}} \cdot \overbrace{C_{4,1} \cdot C_{4,1} \cdot C_{4,1}}^{\textit{pick}\text{ the suits}}}{C_{52,5}}$$

$$P(One\ pair) = \frac{1,098,240}{2,598,960} \approx 0.4225690276$$

$$Odds(One\ pair) \approx 2 \text{ in } 5$$

No pair – 5 different ranks each of any suit



$$P(No\ pair) = \frac{\overbrace{C_{13,5}}^{pick\ \text{5 ranks}} \cdot \overbrace{C_{4,1} \cdot C_{4,1} \cdot C_{4,1} \cdot C_{4,1} \cdot C_{4,1}}^{pick\ \text{each suit}} - \overbrace{4}^{royal} - \overbrace{36}^{straight\ flushes} - \overbrace{5{,}108}^{flushes} - \overbrace{10{,}200}^{straights}}{C_{52,5}}$$

$$P(No\ pair) = \frac{1{,}302{,}540}{2{,}598{,}960} \approx 0.5011773940$$

$$Odds(No\ pair) \approx 1 \text{ in } 2$$

5-Card Poker Hand Summary

Hand	How many	Probability	Odds
Royal Flush	4	0.000001539	1 in 649,740
Straight Flush	36	0.000013857	1 in 72,193
4 of a Kind	624	0.000240096	1 in 4,165
Full House	3,744	0.001440576	1 in 694
Flush	5,108	0.001965402	1 in 509
Straight	10,200	0.003924647	1 in 255
3 of a Kind	54,912	0.021128451	1 in 47
Two Pair	123,552	0.047539016	1 in 21
One Pair	1,098,240	0.422569028	2 in 5
No Pair	1,302,540	0.501177394	1 in 2
	2,598,960		