

MATH 313 Spring 2009, Homework 11, Computer-based problems

Symmetric matrices show up in analytic geometry in the definition of higher-dimensional quadratic surfaces. In general, if A is a symmetric matrix, then the set given by $\{x \in \mathbb{R}^n \mid x^T A x = c\}$ for some fixed $c > 0$ is a quadratic surface.

Problem 1 If A is positive definite, show that the surface defined by $x^T A x = c$ is an ellipsoid. What defines its principal axes? What happens if A has some negative eigenvalues?

Problem 2 Compute the volume of the region enclosed by the surface $x^T B x = 2$ for

$$B = \begin{pmatrix} 1010 & -79 & 155 & 5 & 9 \\ -79 & 712 & 170 & -157 & 59 \\ 155 & 170 & 1007 & 116 & -16 \\ 5 & -157 & 116 & 934 & -16 \\ 9 & 59 & -16 & -16 & 828 \end{pmatrix},$$

using the Matlab command “eig(B)” to compute its eigenvalue decomposition. List its principal axes as well.