

MATH 513 Spring 2009, Homework 8, Addendum

This addendum is a bit more abstract than the previous assignments. We look at vector space quotients, which are useful in various contexts.

Suppose we have a vector space V and a subspace $W \subset V$. The *quotient* V/W is the vector space that's formed by elements of the form $v + W$, where $v \in V$. That is, the elements of the new vector space V/W are shifted copies of W !

Problem 1 Show that V/W is a vector space. In particular, you'll need to tell me how to add two elements $v + W$ and $z + W$, and how to scalar multiply them too. Which element plays the role of zero?

Once you know that V/W is a vector space, we can do all sorts of fun things with it... For instance

Problem 2 What is the dimension of V/W (in terms the dimensions of V and W)?

Quotient spaces are related to orthogonality in a nice way. Suppose $\{w_1, w_2, \dots\}$ is an orthonormal basis for W . The map $p : V \rightarrow V$ given by

$$p(v) = v - \sum_k w_k w_k^T v \tag{1}$$

is a projection map.

Problem 3 What is the dimension of the kernel of the map p above? The image? How does these relate to your answer for Problem 2? (Hint: p can be thought of as an orthogonal projection of V to V/W .)