

## Problems on Change of Variables

Given that for  $a, b > 0$  we have

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(ax^2+by^2)} dx dy = \pi(ab)^{-1/2}$$

1. Compute

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(4x^2+4xy+4y^2)} dx dy$$

$$\text{Hint } 4x^2+4xy+4y^2 = 3(x+y)^2 + (x-y)^2$$

2. Compute

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(2x^2+2xy+y^2)} dx dy$$

$$\text{Hint } 2x^2+2xy+y^2 = (x+y)^2 + x^2$$

3. Compute

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(2x^2+2x+y^2)} dx dy$$

$$\text{Hint } 2x^2+2x+y^2 = 2(x+\frac{1}{2})^2 + y^2 - \frac{1}{2}$$

4. (Like one in book 16.9) Compute

$$\iint_R e^{(x+y)/(x-y)} dA \text{ over the trapizoid with vertices}$$
$$(x,y) = (1,0), (4,0) (0,-1) (0,-4)$$

Answers.  $\sqrt{3}\pi/6$ ,  $\pi$ ,  $\pi\sqrt{2e}/2$ ,  $15(e-1/e)/4$