

**Math 114 (Powers) 1½ Hour Test. Thursday February 12, 2009**

Name(print) \_\_\_\_\_ Penn I.D. \_\_\_\_\_

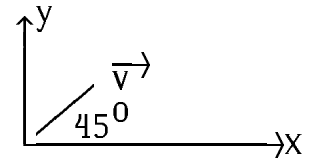
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- The function  $y(t)$  satisfies the differential equation  $\frac{dy}{dt} = ky$  where  $k$  is a constant. If  $y(0) = 2$ ,  $y(2) = 8$  then what is  $y(3)$ ?  $y(3) =$   
 A.  $e^2$  B.  $2e$  C.  $2e^{-1/2}$  D.  $10$  E.  $16$  F.  $2e^2$  G.  $8\sqrt{2}$  H.  $24$
- The function  $y(t)$  satisfies the differential equation  $\frac{dy}{dt} = 3y(2 - y)$ . If  $y(0) = 1$  then as  $t \rightarrow +\infty$   $y(t)$  approaches what?  $y(t) \rightarrow$   
 A.  $+\infty$  B.  $0$  C.  $1/2$  D.  $1/3$  E.  $2$  F.  $3$  G.  $6$  H.  $4\frac{1}{2}$
- Suppose  $y$  satisfies the differential equation  $\frac{dy}{dx} = -xy$  and  $y(0) = 2$ . Then what is  $y(2)$ .  $y(2) =$   
 A.  $2e$  B.  $2e^{-2}$  C.  $2e^{-1}$  D.  $e$  F.  $4\sqrt{2}$  G.  $2\sqrt{2}$  H.  $\sqrt{2}$
- Suppose  $y(t)$  satisfies the differential equations  $(4+t)\frac{dy}{dt} + y = 4 + t$  and  $y(0) = 1$ . Then what is  $y(2)$ ?  $y(2) =$   
 A.  $0$  B.  $e$  C.  $7/3$  D.  $5/3$  F.  $-1$  G.  $11/2$  H.  $\sqrt{2}$
- Suppose  $y(x)$  satisfies the integral equation  $y(x) = 2 + \int_0^x \frac{1}{2}y(t) dt$ . Then what is  $y(2)$ ? (Hint. Differentiate)  $y(2) =$   
 A.  $2e^{1/2}$  B.  $2/e$  C.  $2$  D.  $2e^2$  E.  $2e$  F.  $2\sqrt{2}$  G.  $4$  H.  $8/3$
- Find the distance from the plane  $3x + 2y + z = 14$  to the origin. Hint, find the point on the plane closest to the origin and find its distance from the origin. Distance =  
 A.  $1$  B.  $\sqrt{6}$  C.  $\sqrt{11}$  D.  $\sqrt{14}$  E.  $4$  F.  $\sqrt{18}$  G.  $2\sqrt{5}$  H.  $6$
- Find where the plane through  $\langle 2,1,1 \rangle$ ,  $\langle 1,2,1 \rangle$  and  $\langle 1,1,2 \rangle$  intersects the  $x$ -axis.  $x$ -intersect =  
 A.  $-2$  B.  $-1$  C.  $0$  D.  $1$  E.  $2$  F.  $3$  G.  $4$  H.  $5$
- Find where the straight line through  $\langle 1,0,1 \rangle$  and  $\langle 0,-1,0 \rangle$  passes through the plane  $x + y + z = 5$ . What is the  $x$ -coordinate of the point?  
 A.  $-2$  B.  $-1$  C.  $0$  D.  $1$  E.  $2$  F.  $3$  G.  $4$  H.  $5$

9. Find the plane equidistant from the points  $\langle 1, 3, 0 \rangle$  and  $\langle -1, 1, 2 \rangle$ . Where does this plane intersect the x-axis? x-intersect =

- A. -2 B. -1 C. 0 D. 1~~##~~ E. 2 F. 3 G. 4 H. 5

10. A ball is fired at an angle of  $45^\circ$  with a speed of 48 feet per second. (i.e.  $|\vec{v}| = 48$  ft/sec.) The initial position is  $\langle 0, 0 \rangle$  and the acceleration is constant and directed downward  $\vec{a} = (0, -g)$  where  $g = 32$  ft/sec<sup>2</sup>.



How many feet does the ball travel before it strikes the ground (i.e.  $y = 0$ )

- A. 16 B. 24 C. 32 D. 48 E. 64 F. 72~~##~~ G. 96 H. 128

11. Find the arc length of the curve  $(t\sqrt{2}, e^t, e^{-t})$  from  $t = 0$  to  $t = 2$ .

- A.  $2e^2$  B.  $e^2 - e^{-2}$ ~~##~~ C.  $e + 1/e$  D.  $2\ln(2)$  E.  $4\sqrt{2}$  F.  $e^{1/2}$  G.  $\sqrt{2}(e-1)$

12. As  $\theta$  goes from 0 to  $2\pi$  the point  $(a \cdot \cos(\theta), b \cdot \sin(\theta))$  goes around the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . Find the curvature of the ellipse at  $x = 0, y = b$  or when  $\theta = \pi/2$  ( $90^\circ$ ). Curvature =

- A.  $\frac{a}{a^2 + b^2}$  B.  $\frac{2a}{a^2 + b^2}$  C.  $\frac{b}{a^2 + b^2}$  D.  $\frac{2b}{a^2 + b^2}$  E.  $\frac{b}{a^2}$ ~~##~~ F.  $\frac{a}{b^2}$  G.  $\frac{1}{a}$  H.  $\frac{1}{b}$

13. Consider the curve  $\langle t, t^2, (2/3)t^3 \rangle$ . Find the curvature at  $t = 1$ .

- A. 0 B. 1 C. 2 D.  $2/3$  E.  $4/3$  F.  $2/9$ ~~##~~ G.  $2/5$  H.  $3/11$