Math 114 (Powers) 1½ Hour Test. Thursday April 2, 2009

1. Find the distance between the straight lines, $\vec{r_1}(t) = \langle 1+2t, 1, 2+2t \rangle$ and $\vec{r_2}(t) = \langle t, -1, 2t \rangle$ where t runs from $-\infty$ to $+\infty$.

   A. 1/3 B. $1/\sqrt{3}$ C. 1/2 D. 2/3 E. $2/\sqrt{3}$ F. 2 ## G. $\sqrt{2}$ H. $\sqrt{5}$

2. Suppose y(x) satisfies the intergral equation $y(x) = 2 - \int_0^x y(t)\,dt$. Then what is y(1)? (Hint. Differentiate) y(1) =

   A. $2e^{1/2}$ B. $2/e$ ## C. 2 D. $2e^2$ E. 2e F. $2\sqrt{2}$ G. 4 H. 8/3

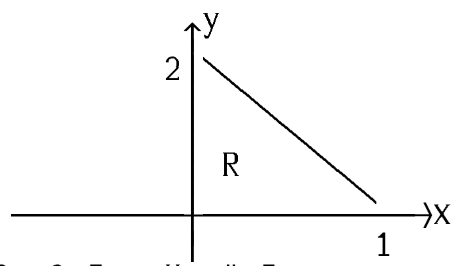
3. The curve $\vec{r}(t) = (4\cos(t), 4\sin(t), 3t)$ has constant curvature. What is the curvature.

   A. 0 B. 1/15 C. 1/8 D. 4/25 ## E. 7/25 F. 3/8 G. $\frac{1}{2}\sqrt{2}$ H. $\frac{1}{2}\sqrt{3}$

4. Find the maximum of $f(x,y,z) = x + 2y + 4z$ on the elipsoid $x^2 + y^2 + 2z^2 = 13$

   A. 0 B. 4 C. 7 D. 10 E. 13 ## F. 15 G. 16 H. 20

5. Let R be the region inside the triange with vertices (x=0,y=0), (x=1,y=0) and (x=0,y=2). Let f(x,y) = y. Then $\iint_R f(x,y)\,dA$



   A. 0 B. 1/3 C. 2 D. 1 E. 1/2 F. 1/6 G. 2/3 ## H. 4/3

6. Evaluate $\int_0^4 \int_{\frac{1}{2}y}^2 e^{\frac{1}{2}x^2}\,dx\,dy$. (Hint. You will need to exchange the order of integration.)

   A. 0 B. e^{-2} C. e + 1 D. $2(e^2 - 1)$ ## E. 4ln(2) - 1 F. $e^4 - 2$ G. $\frac{1}{2}(e^8 - 1)$ H. 4

7. The probability density of finding point at (x,y,z) in the unit cube C = $\{0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1\}$ is $f(x,y,z) = y + z$. What is the probability that z is less than ½. Note $\iiint_C f(x,y,z)\,dV = 1$

   $\text{Prob}(0 \le z \le \frac{1}{2}) =$

   A. 1/16 B. 1/8 C. 3/16 D. 1/4 E. 5/16 F. 3/8 ## G. 7/16 H. 1/2

8. Find the area of the region inside the curve given in polar coordinates by $r(\theta) = 2 + \sin(4\theta)$ where $-\pi < \theta \leq \pi$.

A. $3\pi/2$ B. $5\pi/3$ C. 3π D. $\sqrt{10}\,\pi$ E. $9\pi/2$ ## F. $3\sqrt{5}\,\pi$ G. 15π H. $3\sqrt{7}\,\pi$

9. Compute the integral of x over the region $R = \{x^2 + y^2 \leq 9 \text{ and } x \geq 0\}$.

i.e. compute $\iint_R x \, dA$

A. 6 B. $4\sqrt{3}$ C. 3π D. 11 E. $5\sqrt{5}$ F. 18 ## G. 6π H. 21

10. A sphere six inches in diameter ($r = 3$) is filled with water so the water is two inches deep. (i.e. if $z = -3$ is the bottom of the sphere $z = -1$ at the surface of the water.). How many cubic inches of water in the sphere. V =

A. $\frac{10\pi}{3}$ B. 4π C. $\frac{13\pi}{3}$ D. 5π E. 16 F. $\frac{23\pi}{3}$ G. $\frac{28\pi}{3}$ ## H. 32

11. Compute z-component of the the center of mass of region above the parabolic surface $z = x^2 + y^2$ and below $z = 2$ (i.e. $R = x^2 + y^2 \leq z \leq 2$.)
Compute $(0,0,\bar{z}) = \frac{1}{V}\iiint_R \vec{r}\,dV$ where $V = \iiint_R dV = 2\pi$. Ans $\bar{z}$ =

A. 2/3 B. 1 C. 4/3 ## D. 7/5 E. 3/2 F. 5/3 G. 7/4 H. $\sqrt{2}$

12. Compute moment inertia of material of unit density in region above the parabolic surface $z = x^2 + y^2$ and below $z = 2$ (i.e. $R = x^2 + y^2 \leq z \leq 2$.)
Compute $I_z = \iiint_R x^2+y^2 \, dV$ Ans I_z =

A. $4\pi/3$ ## B. $5\pi/3$ C. 2π D. $7\pi/3$ E. $8\pi/3$ F. 3π G. $10\pi/3$ H. 4π