Math 240: Flux and Stokes' Theorem

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Thursday March 24, 2011

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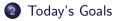
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Outline









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Review of Last Time

- Learned how to find surface area.
- Learned how to set up and evaluate surface integrals.
- S Learned what an orientable surface is.

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Review

Orientations

Definition

An **orientable** surface has two sides that can be painted red and blue resp.

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Theorem

If a surface is given by g(x, y, z) = 0 then the unit normals are given by $\mathbf{n} = \frac{\pm 1}{||\nabla g||} \nabla g$

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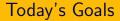
Theorem

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Exercise Find the unit normal vectors to a sphere of radius *a*.

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- Be able to set up and evaluate flux integrals.
- **2** Understand when and how to use Stokes' theorem.

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Flux

Integrals of vector fields

Suppose $F(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$ models the velocity of a fluid in 3-space.

Flux

Integrals of vector fields

Suppose $F(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$ models the velocity of a fluid in 3-space.

Then the volume flowing through a small patch of surface S per unit time is

 $(F \circ \mathbf{n})\Delta S.$

Where **n** is the normal vector to *S* and ΔS is the area of the patch of surface.

Flux

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The total volume per unit time is the Flux and is given by

$$\int \int_{S} (F \circ \mathbf{n}) dS$$

Example Let $T(x, y, z) = x^2 + y^2 + z^2$ model a temperature distribution in 3-space. From physics, heat flow is modeled by $F = -\nabla T$. Find the heat flow out of a sphere of radius *a* centered at the origin.

Flux

Given a curve in 3-space C: x = f(t), y = g(t), z = h(t). The position vector of C is $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$. $d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k} = f'(t)dt\mathbf{i} + g'(t)dt\mathbf{j} + h'(t)dt\mathbf{k} = \mathbf{T}dt$ Where **T** is tangent vector to C.

Stokes' Theorem

Theorem

Let S be a piecewise smooth orientable surface bounded by a piecewise smooth simple closed curve C. Let $F(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$ be a vector field for which P, Q and R are continuous and have continuous partial derivatives in the region of 3-space containing S. If C is traversed in the positive direction and \mathbf{T} is the unit tangent vector to C then

$$\oint_C F \circ d\mathbf{r} = \oint_C (F \circ \mathbf{T}) ds = \int \int_S (curl(F) \circ \mathbf{n}) dS$$

where **n** is the unit normal to S in the direction of the orientation of S.

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Example: Use Stokes' theorem to evaluate $\oint_C F \circ d\mathbf{r}$ where *C* is the triangle with vertices (1, 0, 0), (0, 1, 0) and (0, 0, 1) oriented counterclockwise when viewed from above and $F = (2z + x)\mathbf{i} + (y - z)\mathbf{j} + (x + y)\mathbf{k}$.