Math 240: The Divergence Theorem

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Thursday March 31, 2011

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Review

Review of Last Time

Solved flux integrals.

Learned about Stokes' theorem and its uses.

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Stokes' Theorem

Theorem

Let S be a piecewise smooth orientable surface bounded by a piecewise smooth simple closed curve C. Let $F(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$ be a vector field for which P, Q and R are continuous and have continuous partial derivatives in the region of 3-space containing S. If C is traversed in the positive direction and **T** is the unit tangent vector to C then

$$\oint_C F \circ d\mathbf{r} = \oint_C (F \circ \mathbf{T}) ds = \int \int_S (curl(F) \circ \mathbf{n}) dS$$

where **n** is the unit normal to S in the direction of the orientation of S.

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Stokes' Theorem

Theorem

Let S be a nice surface bounded by a nice curve C. Let F be a nice vector field. If C is traversed in the positive direction and \mathbf{T} is the unit tangent vector to C then

$$\oint_C F \circ d\mathbf{r} = \oint_C (F \circ \mathbf{T}) ds = \int \int_S (curl(F) \circ \mathbf{n}) dS$$

where **n** is the unit normal to S in the direction of the orientation of S.

Stokes' Theorem

Theorem

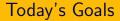
Let S be a nice surface bounded by a nice curve C. Let F be a nice vector field. If C is traversed in the positive direction and \mathbf{T} is the unit tangent vector to C then

$$\oint_C F \circ d\mathbf{r} = \oint_C (F \circ \mathbf{T}) ds = \int \int_S (curl(F) \circ \mathbf{n}) dS$$

where **n** is the unit normal to S in the direction of the orientation of S.

Let $F = \langle y^2, 2z + x, 2y^2 \rangle$. Find a plane ax + by + cz = 0 such that $\oint_C F \circ dr = 0$ for every closed curve C in the plane.

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• Understand when and how to use the divergence theorem.

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Divergence Theorem

Theorem

Let D be a nice region in 3-space with nice boundary S oriented outward. Let F be a nice vector field. Then

$$\int \int_{S} (F \circ \mathbf{n}) dS = \int \int \int_{D} div(F) dV$$

where **n** is the unit normal vector to S.

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Theorem

Let D be a closed and bounded region in 3-space with a piecewise smooth boundary S that is oriented outward. Let $F(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$ be a vector field for which P, Q and R are continuous and have continuous first partial derivatives in a region of 3-space containing D. Then

$$\int \int_{S} (F \circ \mathbf{n}) dS = \int \int \int_{D} div(F) dV$$

where \mathbf{n} is the unit normal vector to S.

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