## Math 240: Phase Portraits

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Tuesday April 12, 2011

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Math 240: Phase Portraits

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### Review of Last Time

#### • Learned how to solve constant coefficient systems.

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#### Solutions to 2 by 2 systems

General Solution 2 by 2 System with Distinct Real Eigenvalues  $\lambda_1$  and  $\lambda_2$ :

$$\mathbf{X} = c_1 \mathbf{K}_1 e^{\lambda_1 t} + c_2 \mathbf{K}_2 e^{\lambda_2 t}$$

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General Solution 2 by 2 System with Repeated Real Eigenvalue  $\lambda$ :

$$c_1 \mathbf{K} e^{\lambda t} + [\mathbf{K} t e^{\lambda t} + \mathbf{P} e^{\lambda t}]$$

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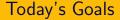
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General Solution 2 by 2 System with Complex Eigenvalue  $\alpha + i\beta$ :

$$c_1[Re(\mathbf{K})cos(eta t) - Im(\mathbf{K})sin(eta t)]e^{lpha t}$$

$$+c_2[Im(\mathbf{K})cos(eta t)+Re(\mathbf{K})sin(eta t)]e^{lpha t}$$



#### • Use phase portrait to develop intuition for systems.

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#### Guessing a Solution

# A solution to a 2 by 2 initial value system gives a curve $r(t) = \langle x(t), y(t) \rangle$ in the xy-plane (the **Phase Plane**).

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These solution curves have tangent vectors given by the vector field  $F = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j}$  (the **Prase Portrait**)

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We can gain qualitative and quantitative information about a system by looking at its Phase Portrait.

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