Math 240: Solving Systems of DEs by Diagonalization

Ryan Blair

University of Pennsylvania

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Ryan Blair (U Penn)

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Outline





- Oiagonalization and Systems
- 4 Review of Power Series

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Review

Review of Last Time

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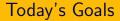
 Used phase portraits to make qualitative and quantitative statements about systems.

Review

Solutions to 2 by 2 systems

- Two linear solutions in the phase plane implies two distinct real eigenvalues.
- One linear solution in the phase plane implies one repeated real eigenvalue.
- Solutions implies complex eigenvalues.

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- Use diagonalization to solve systems of linear DEs.
- Review power series.

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Diagonalization and Systems

Coupled and Uncoupled Systems

Definition

A system $\mathbf{X}' = \mathbf{A}\mathbf{X}$ is **coupled** if each x'_i is expressed as a linear combination of $x_1, ..., x_i$. If \mathbf{A} is diagnolizable, then the system can be uncoupled, in that each x'_i can be expressed solely in terms of x_i .

Recall the following theorem from Linear Algebra

Theorem

If an $n \times n$ matrix A has n linearly independent eigenvectors, then A is diagnolizable.

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Given a system $\mathbf{X}' = \mathbf{A}\mathbf{X}$. Suppose **A** is diagnolizable(i.e. $P^{-1}AP = D$).

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If an $n \times n$ matrix A has n linearly independent eigenvectors, then A is diagnolizable.

Given a system $\mathbf{X}' = \mathbf{A}\mathbf{X}$. Suppose \mathbf{A} is diagnolizable(i.e. $P^{-1}AP = D$). If $\mathbf{X} = \mathbf{P}\mathbf{Y}$ then

$$\mathbf{X} = \mathbf{P} \begin{bmatrix} c_1 e^{\lambda_1 t} \\ \vdots \\ c_n e^{\lambda_n t} \end{bmatrix}$$

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Review of Power Series

Definition

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1 (x-a) + c_2 (x-a)^2 + \dots$$

is a power series centered at a.

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Definition

The **radius of convergence** is the largest R such that $\sum_{n=o}^{\infty} c_n (x-a)^n$ converges for all x such that |x-a| < R.

Finding the Radius of Convergence

Ratio Test

Let

$$\lim_{n \to \infty} |\frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n}| = |x-a|\lim_{n \to \infty} |\frac{c_{n+1}}{c_n}| = L$$

If L < 1 the series converges. If L > 1 the series diverges. If L = 1 we don't know.

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Review of Power Series

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