Quiz 1

NAME:____________________________________

For your own benefit show all of your work, keep it neat, and if you write more than one solution make clear which is your answer or I will just grade whichever one is closest to the top of the paper.

1. (10pts) Suppose $A = \begin{pmatrix} 5 & 2 & 4 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Then:

a. (3pts) Find the rank of A
b. (2pts) Find the determinant of A
c. (5pts) Find A^{-1}, if it exists

a. $\begin{pmatrix} 5 & 2 & 4 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} R_1 \mapsto \frac{1}{5}R_1 \begin{pmatrix} 1 & \frac{2}{5} & \frac{4}{5} \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} R_2 \mapsto R_2 - 3R_1 \begin{pmatrix} 1 & \frac{2}{5} & \frac{4}{5} \\ 0 & -\frac{1}{5} & -\frac{12}{5} \\ 0 & 0 & 1 \end{pmatrix}$ $R_2 \mapsto -5R_2 \begin{pmatrix} 1 & \frac{2}{5} & \frac{4}{5} \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{pmatrix}$ Thus A is rank 3. Another acceptable answer would be since $\det(A) \neq 0$, A must have rank 3 from what we said in class.

b. $\begin{vmatrix} 5 & 2 & 4 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 5 & 2 \\ 3 & 1 \end{vmatrix} = -1$ (expanding along the bottom row)

c. Since $\det(A) \neq 0$, we know that A^{-1} exists. We compute it by using the augmented matrix $(A|I)$:

$$\left(\begin{array}{ccc|ccc} 5 & 2 & 4 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) R_1 \mapsto R_1 - 2R_2$$

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 4 & 1 & -2 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) R_1 \mapsto R_1 - 4R_3$$

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 0 & 1 & -2 & -4 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) R_2 \mapsto R_2 + 3R_1$$

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 0 & 1 & -2 & -4 \\ 0 & 1 & 0 & 3 & -5 & -12 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) R_1 \mapsto -R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 4 \\ 0 & 1 & 0 & 3 & -5 & -12 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

Thus we have that $A^{-1} = \begin{pmatrix} -1 & 2 & 4 \\ 3 & -5 & -12 \\ 0 & 0 & 1 \end{pmatrix}$

2. (10pts) Find the eigenvalues and eigenvectors of the matrix:

$$A = \begin{pmatrix} -3 & 9 \\ -4 & 9 \end{pmatrix}$$

Can A be diagonalized?
First we find the eigenvalues (4pts):

$$\begin{aligned} \begin{vmatrix} -3-\lambda & 9 \\ -4 & 9-\lambda \end{vmatrix} &= (-3-\lambda)(9-\lambda) + 36 \\ &= \lambda^2 - 9\lambda + 3\lambda - 27 + 36 \\ &= \lambda^2 - 6\lambda + 9 \\ &= (\lambda - 3)^2 = 0 \end{aligned}$$

So we have that $\lambda = 3, 3$. Now we check for the eigenvectors for this eigenvalue (4pts):
Look at the system $(A - \lambda I)v = 0$ for $\lambda = 3$:

$$\left(\begin{array}{cc|c} -6 & 9 & 0 \\ -4 & 6 & 0 \end{array} \right) R_2 \mapsto R_2 - \frac{2}{3} R_1 \left(\begin{array}{cc|c} -6 & 9 & 0 \\ 0 & 0 & 0 \end{array} \right)$$

So we have only one parameter, and thus one linearly independent eigenvector. Our condition is that $-6x + 9y = 0$, or $3y = 2x$. So we could choose for example the vector $v = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Since there is only one linearly independent eigenvector for the matrix, it cannot be diagonalized (2pts).