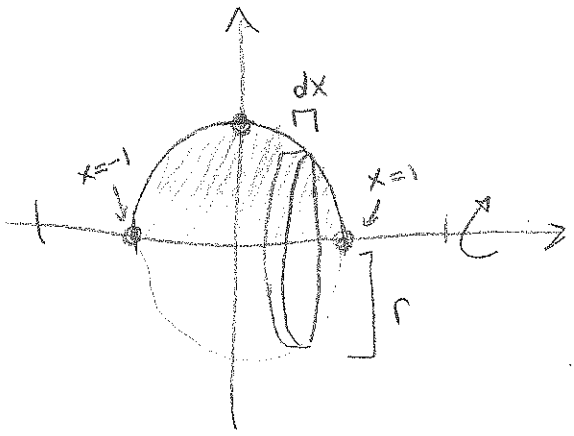
Quiz 1

NAME:_____

RECITATION : Mon8 Mon9 Wed8 Wed9

1. Find the volume of revolution defined by rotating the region below $y = 1 - x^2$ and above $y = 0$ around the x-axis.



$r = y = 1 - x^2$

$$V = \int_{-1}^{1} \pi r^2 \, dx$$

$$= \int_{-1}^{1} \pi (1-x^2)^2 \, dx$$

$$= \pi \int_{-1}^{1} x^4 - 2x^2 + 1 \, dx$$

$$= \pi \left[\frac{x^5}{5} - \frac{2}{3}x^3 + x \right]_{-1}^{1}$$

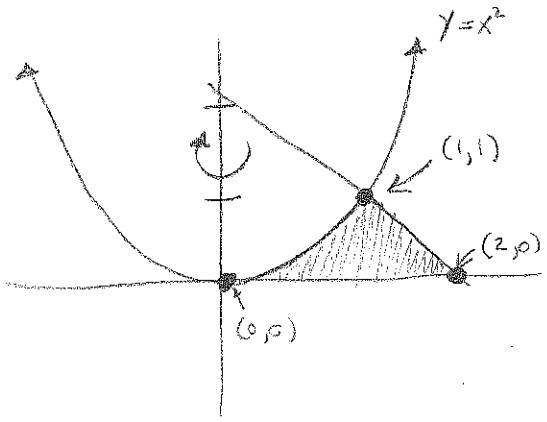
$$= \pi \left(\left[\frac{1}{5} - \frac{2}{3} + 1 \right] - \left[-\frac{1}{5} + \frac{2}{3} - 1 \right] \right)$$

$$= \pi \left(\frac{2}{5} - \frac{4}{3} + 2 \right)$$

$$= \pi \left(\frac{6}{15} - \frac{20}{15} + \frac{30}{15} \right)$$

$$= \frac{16\pi}{15}$$

2. Set up (only!) an equation for the volume of revolution defined by rotating the region D around the y-axis, where D is the region below $y = x^2$, below $y = 2 - x$, and with both $x, y \geq 0$.



WASHERS:

$$V = \pi \int_0^1 r_{OUT}^2 - r_{IN}^2 \, dy$$

$$= \pi \int_0^1 \left[(2-y)^2 - y \right] dy$$

$r_{IN} = x$ ALONG $y = x^2$

$r_{IN} = \sqrt{y}$

$r_{OUT} = x$ ALONG $y = 2 - x$

$r_{OUT} = 2 - y$

OR SHELLS:

$$V = 2\pi \int_0^1 rh \, dx + 2\pi \int_1^2 rh \, dx$$

$$= 2\pi \int_0^1 x(x^2) \, dx + 2\pi \int_1^2 x(2-x) \, dx$$