Monday

Quiz 2

NAME:__________________________

RECITATION : Mon8 Mon9 Wed8 Wed9

1. Find the length of arc of the curve $y = \frac{2}{3}x^{3/2}$ for $0 \le x \le 3$.

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \left(\frac{2}{3} \cdot \frac{3}{2}\right) x^{1/2} = x^{1/2} = \sqrt{x}$$

$$\left(\frac{dy}{dx}\right)^2 = \left(x^{1/2}\right)^2 = \left(\sqrt{x}\right)^2 = x$$

$$\Rightarrow \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + x}$$

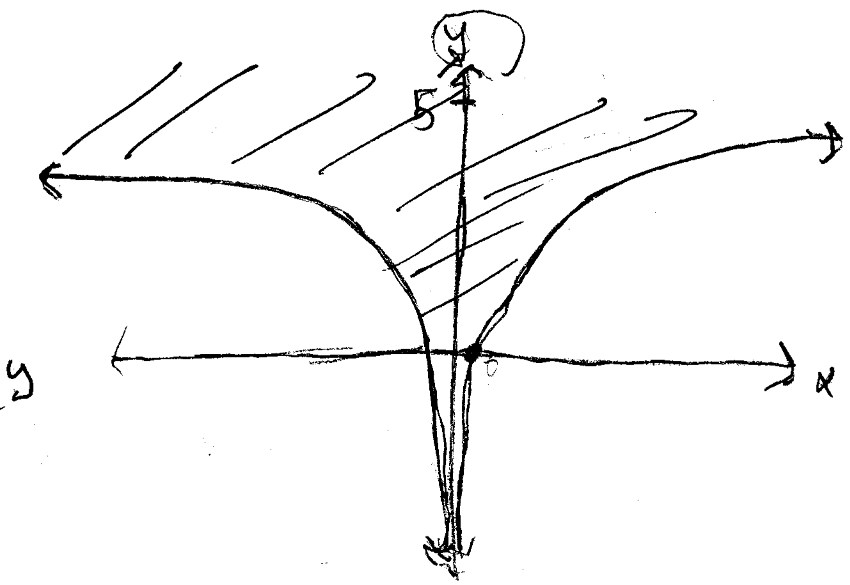
$$\Rightarrow L = \int_0^3 \sqrt{x+1}\, dx$$

$$= \frac{2}{3}(x+1)^{3/2} \Big|_0^3 = \frac{2}{3}\left[\left(4^{3/2}\right) - \left(1^{3/2}\right)\right]$$

$$= \frac{2}{3}\left[8 - 1\right] = \boxed{\frac{14}{3}}$$

2. Set up (only!) an equation for the surface area of the region generated by rotating the curve $y = \ln(4x)$ for $0 \le y \le 5$ about the y-axis.

$$SA = \int_{y_1}^{y_2} 2\pi\, g(y) \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$



$$y = \ln(4x)$$

$$\Rightarrow e^y = e^{\ln(4x)} = 4x$$

$$\Rightarrow x = g(y) = \frac{1}{4}e^y$$

$$\left(\frac{dx}{dy}\right) = \frac{1}{4}e^y$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{16}e^{2y}$$

$$SA = \int_0^5 2\pi\left(\frac{1}{4}e^y\right)\sqrt{1 + \frac{1}{16}e^{2y}}\, dy$$

$$= \frac{\pi}{2}\int_0^5 e^y\sqrt{1 + \frac{1}{16}e^{2y}}\, dy$$