Wednesday

Quiz 2

NAME:_________________________

RECITATION : Mon8 Mon9 Wed8 Wed9

1. Find the length of arc of the curve $y = \frac{1}{3}(x^2+2)^{3/2}$ for $0 \le x \le 3$.

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \left(\frac{1}{3}\cdot\frac{3}{2}\right)(x^2+2)^{1/2}\cdot(2x)$$

$$= x(x^2+2)^{1/2} = x\sqrt{x^2+2}$$

$$\left(\frac{dy}{dx}\right)^2 = x^2(x^2+2) = x^4+2x^2$$

$$\Rightarrow \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{x^4+2x^2+1} = \sqrt{(x^2+1)^2} = x^2+1$$

$$L = \int_0^3 \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \int_0^3 x^2+1\, dx$$

$$= \left[\frac{x^3}{3}+x\right]_0^3 = \left[(9+3)-(0+0)\right]$$

$$= 12$$

2. Set up (only!) an equation for the surface area of the region generated by rotating the curve $y = e^{4x}$ for $0 \leq y \leq 5$ about the y-axis.

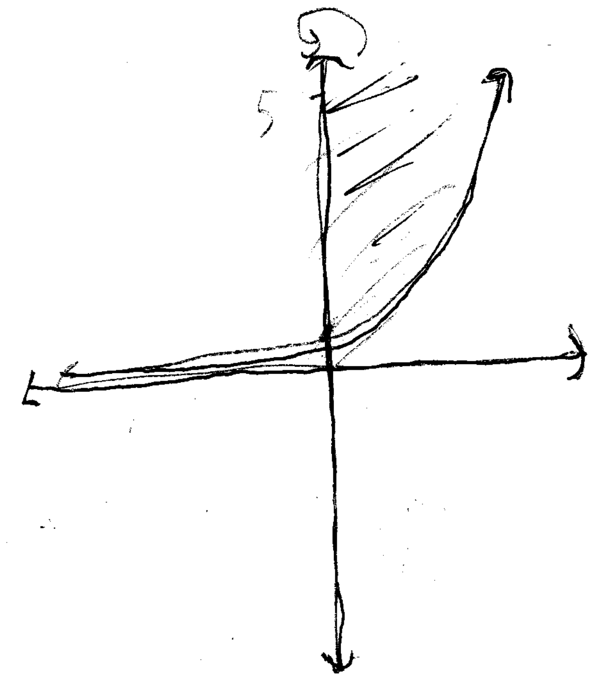
$$SA = \int_{y_1}^{y_2} 2\pi\, g(y) \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$y = e^{4x}$$

$$\Rightarrow \ln(y) = \ln(e^{4x})$$

$$\Rightarrow 4x = \ln(y)$$

$$\Rightarrow x = \frac{1}{4}\ln(y) = g(y)$$



$$\left(\frac{dx}{dy}\right) = \frac{1}{4y}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{16y^2}$$

$$SA = \int_0^5 2\pi \cdot \left(\frac{1}{4}|\ln(y)|\right) \sqrt{1 + \frac{1}{16y^2}}\, dy$$

$$= \frac{\pi}{2} \int_0^5 |\ln(y)| \sqrt{1 + \frac{1}{16y^2}}\, dy$$

Note:

I made a mistake with the bounds. Should have been $1 \leq y \leq 5$ because $y = e^{4x}$ never touches $y = 0$.

But if set up correctly (as if $y=0$ works) then correct.