M

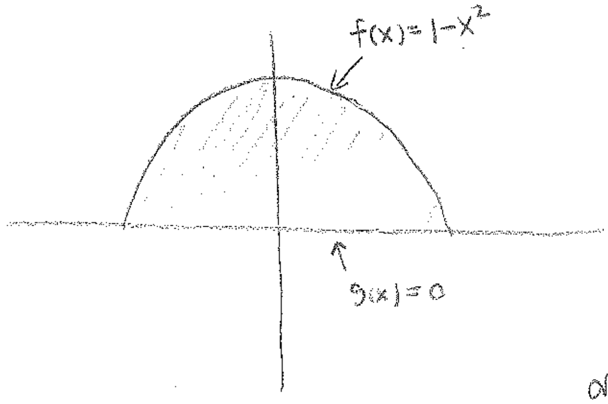
Quiz 3

NAME:_________________________

RECITATION : Mon8 Mon9 Wed8 Wed9

1. Find the center of mass $(\bar{x}, \bar{y})$ of the region below $y = 1 - x^2$ and above $y = 0$, assuming that the density of the region is constant.

$\bar{X} = 0$ (SEE PIC)



$$\bar{Y} = \frac{M_x}{M}$$

$$M_x = \frac{1}{2}\int_{-1}^{1} f(x)^2 - g(x)^2\,dx$$

$$= \frac{1}{2}\int (1-x^2)^2\,dx$$

OR NOTICE SYMMETRY

$$= \frac{1}{2}\int x^4 - 2x^2 + 1\,dx$$

$$= \frac{1}{2}\left[\frac{1}{5}x^5 - \frac{2}{3}x^3 + x\right]\Bigg|_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{1}{5} - \frac{2}{3} + 1\right] - \frac{1}{2}\left[-\frac{1}{5} + \frac{2}{3} - 1\right]$$

$$= \frac{1}{2}\left[\frac{3}{15} - \frac{10}{15} + \frac{15}{15}\right] - \frac{1}{2}\left[-\frac{3}{15} + \frac{10}{15} - \frac{15}{15}\right]$$

$$= \frac{1}{2}\left[\frac{8}{15}\right] - \frac{1}{2}\left[-\frac{8}{15}\right]$$

$$= \frac{8}{15}$$

$$= \frac{1}{2}\cancel{(2)}\int_0^1 x^4 - 2x^2 + 1\,dx$$

$$= \left[\frac{1}{5}x^5 - \frac{2}{3}x^3 + x\right]\Bigg|_0^1$$

$$= \frac{1}{5} - \frac{2}{3} + 1 = \frac{8}{15}$$

$$M = \int_{-1}^{1} 1 - x^2\,dx \xrightarrow{\text{OR NOTICE SYMMETRY}} = 2\int_0^1 1 - x^2\,dx$$

$$= x - \frac{1}{3}x^3\Bigg|_{-1}^{1}$$

$$= \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)$$

$$= \frac{2}{3} + \frac{2}{3}$$

$$= \frac{4}{3}$$

$$= 2\left[x - \frac{1}{3}x^3\right]\Bigg|_0^1$$

$$= 2\left(\frac{2}{3}\right) = \frac{4}{3}$$

ANYWAYS:

$$\bar{Y} = \frac{\frac{8}{15}}{\frac{4}{3}} = \frac{8}{15}\cdot\frac{3}{4} = \frac{2}{5}$$

$$\left(0, \frac{2}{5}\right)$$

2. Find the anti-derivative:

$$\int \sin^3(x)dx$$

$= \int \sin^2 x \sin x \, dx$

$= \int (1-\cos^2 x) \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$= -\int 1-u^2 \, du$

$= -\left[u - \frac{1}{3}u^3\right] + C$

$= \frac{1}{3}\cos^3 x - \cos x + C$