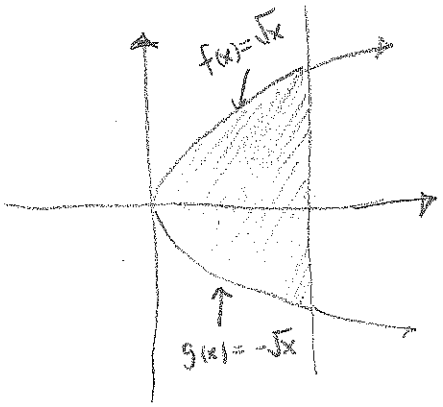
W

Quiz 3

NAME:_____

RECITATION : Mon8 Mon9 Wed8 Wed9

1. Find the center of mass $(\bar{x}, \bar{y})$ of the region between the parabola $x = y^2$ and $x = 1$, assuming that the density of the region is constant.



$\bar{y} = 0$ (see pic)

$\bar{x} = \dfrac{M_y}{M}$

$M = \int_0^1 \sqrt{x} - (-\sqrt{x})\, dx$

$= 2\int \sqrt{x}\, dx$

$= 2\left(\frac{2}{3}\right) x^{3/2} \Big|_0^1$

$= \frac{4}{3}$

$M_y = \int_0^1 x\left(\sqrt{x} - (-\sqrt{x})\right) dx$

$= 2\int x\sqrt{x}\, dx$

$= 2\int x^{3/2}\, dx$

$= 2\left(\frac{2}{5}\right) x^{5/2} \Big|_0^1$

$= \frac{4}{5}$

$\bar{x} = \dfrac{\frac{4}{5}}{\frac{4}{3}} = \frac{4}{5} \cdot \frac{3}{4} = \frac{3}{5}$

$\left(\frac{3}{5}, 0\right)$

2. Find the anti-derivative:

$$\int x \sin(3x)\,dx$$

$u = x$ $\qquad dv = \sin 3x\,dx$

$du = dx$ $\qquad v = -\frac{1}{3}\cos 3x$

$$= -\frac{1}{3}x\cos 3x + \frac{1}{3}\int \cos 3x\,dx$$

$$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C$$