Quiz 4

NAME: Solutions

RECITATION : Mon8 Mon9 Wed8 Wed9

1. Compute $\int_0^1 \frac{2x+3}{x^2-9}dx$

$$\frac{2X+3}{X^2-9} = \frac{2X+3}{(X-3)(X+3)} = \frac{A}{X-3} + \frac{B}{X+3}$$

$$\longrightarrow 2X+3 = A(X+3) + B(X-3)$$

$$2X+3 = AX + 3A + BX - 3B$$

$$2X+3 = (A+B)X + 3(A-B)$$

$$\Rightarrow \left.\begin{array}{l} A+B=2 \\ A-B=1 \end{array}\right\} \text{add: } 2A=3$$

$$\boxed{A = 3/2} \rightarrow \frac{3}{2} + B = 2$$

$$\boxed{B = \frac{1}{2}}$$

$$\int_0^1 \frac{2X+3}{X^2-9}dX = \int_0^1 \frac{\frac{3}{2}}{X-3} + \frac{\frac{1}{2}}{X+3}\,dX$$

$$= \frac{1}{2}\int_0^1 \frac{3}{X-3} + \frac{1}{X+3}\,dX = \frac{1}{2}\Big[3\ln|X-3| + \ln|X+3|\Big]_0^1$$

$$= \frac{1}{2}\Big[\big(3\ln(2) + \ln(4)\big) - \big(3\ln(3) + \ln(3)\big)\Big]$$

$$= \frac{1}{2}\Big(\big(3\ln(2) + 2\ln(2)\big) - \big(4\ln(3)\big)\Big)$$

$$\boxed{= \frac{5}{2}\ln(2) - 2\ln(3)}$$

2. Evaluate the following indefinite integral $\int \frac{\sqrt{9-x^2}}{x^2}dx$

$$x = 3\sin\theta$$
$$dx = 3\cos\theta\, d\theta$$
$$x^2 = 9\sin^2\theta$$

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx = \cancel{\int \frac{\sqrt{9-9\sin^2\theta}}{9\sin^2\theta}} \cancel{\int \frac{3\cos\theta}{9\sin}}$$

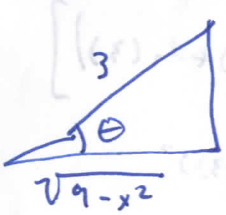
$$\cancel{= \int \sqrt{9-}} \quad = \int \frac{\sqrt{9-9\sin^2\theta}}{9\sin^2\theta} \cdot 3\cos\theta\, d\theta$$

$$= \int \frac{3\cos\theta \cdot 3\cos\theta}{9\sin^2\theta}\,d\theta = \int \frac{\cos^2\theta}{\sin^2\theta}\,d\theta = \int \cot^2\theta\, d\theta$$

Now, $1 + \cot^2\theta = \csc^2\theta$
$\Rightarrow \cot^2\theta = \csc^2\theta - 1$

$$= \int \csc^2\theta - 1\, d\theta = -\cot\theta - \theta + C$$

Now, we look at ref. triangle to convert back:



$$-\cot\theta - \theta + C = \boxed{-\frac{\sqrt{9-x^2}}{x} - \arcsin\left(\frac{x}{3}\right) + C}$$