

NAME: Solutions

Quiz 4

RECITATION: Mon8 Mon9 Wed8 Wed9

1. Compute $\int_0^1 \frac{5x-3}{(x+1)(x-3)} dx$

$$\rightarrow \frac{5x-3}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$\rightarrow 5x-3 = A(x-3) + B(x+1)$$

$$5x-3 = Ax-3A + Bx+B$$

$$5x-3 = (A+B)x - 3A+B$$

$$A+B=5$$

$$-3A+B=-3$$

$$\Rightarrow 4A=8$$

$$A=2$$

$$2+B=5$$

$$\Rightarrow B=3$$

$$\int_0^1 \frac{5x-3}{(x+1)(x-3)} dx = \int_0^1 \frac{2}{x+1} + \frac{3}{x-3} dx$$

$$= 2 \ln|x+1| + 3 \ln|x-3| \Big|_0^1$$

$$= (2 \ln(2) + 3 \ln(2)) - (2 \ln(1) + 3 \ln(3))$$

$$= 5 \ln(2) - 3 \ln(3)$$

2. Evaluate the following indefinite integral $\int \frac{1}{(x^2+1)^2} dx$

$$\int \frac{1}{(x^2+1)^2} dx$$

$$x = \tan \theta$$

$$dx = \sec^2 \theta d\theta$$

$$x^2 = \tan^2 \theta$$

$$\Rightarrow \int \frac{\sec^2 \theta}{(\tan^2 \theta + 1)^2} d\theta = \int \frac{\sec^2 \theta}{\sec^4 \theta} d\theta = \int \frac{1}{\sec^2 \theta} d\theta$$

$$= \int \cos^2 \theta d\theta = \frac{1}{2} \int (1 + \cos(2\theta)) d\theta$$

$$= \frac{1}{2} \left[\theta + \frac{\sin(2\theta)}{2} \right] + C$$

$$= \frac{1}{2} \left[\theta + \sin \theta \cos \theta \right] + C$$

$$= \frac{1}{2} \left[\tan^{-1}(x) + \frac{x}{\sqrt{x^2+1}} \cdot \frac{1}{\sqrt{x^2+1}} \right] + C$$

$$= \frac{1}{2} \left(\tan^{-1}(x) + \frac{x}{x^2+1} \right) + C$$

