MON

Quiz 5

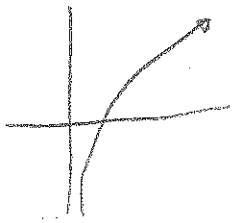
NAME:_____

RECITATION : Mon8 Mon9 Wed8 Wed9

1. Compute the following improper integral or show that it diverges:

$$\int_0^2 \ln(2x)\,dx$$

VERTICAL ASYMPTOTE @ $x=0$



BY PARTS:

$u = \ln(2x)$ $\quad dv = dx$

$du = \frac{1}{2x}(2)\,dx = \frac{dx}{x}$ $\quad v = x$

SO:

$$\int \ln(2x)\,dx = x\ln 2x - \int 1\,dx$$

$$= x\ln 2x - x$$

$$\lim_{t\to 0} \int_t^2 \ln(2x)\,dx = \lim_{t\to 0} \left[x\ln(2x) - x \right]\Big|_t^2$$

$$= \lim_{t\to 0} \left[2\ln 4 - 2 - (t\ln t - t) \right]$$

$t\ln t \to 0$

$2\ln 4 - 2$

$$\lim_{t\to 0} t\ln t$$

$$= \lim_{t\to 0} \frac{\ln t}{\frac{1}{t}} \to \frac{\infty}{\infty}$$

(L'HOP)

$$= \lim_{t\to 0} \frac{\frac{1}{t}}{-\frac{1}{t^2}}$$

$$= \lim_{t\to 0} (-t) = 0$$

2. The function $f(x) = \sec^2(x)$ is a probability density function on the interval $[0, a]$. Find a.

$$\int_0^a \sec^2 x \, dx = 1$$

$$\tan x \Big|_0^a = 1$$

$$\tan a - \tan 0 = 1$$

$$\tan a = 1$$

$$a = \frac{\pi}{4}$$