

WED

Quiz 5

NAME: _____

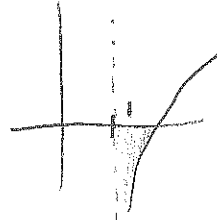
RECITATION : Mon8 Mon9 Wed8 Wed9

1. Compute the following improper integral or show that it diverges:

$$\int_1^2 \frac{1}{1-x^2} dx$$

VERTICAL ASYMPTOTE

@ $x=1$:



PARTIAL FRACTIONS:

$$\frac{1}{1-x^2} = \frac{A}{1-x} + \frac{B}{1+x}$$

$$1 = A(1+x) + B(1-x)$$

$$x=1: 1 = 2A \quad A = \frac{1}{2}$$

$$x=-1: 1 = 2B \quad B = \frac{1}{2}$$

NOTE: $\int \frac{1}{1-x} dx$ $u=1-x$
 $du = -dx$
 $-\int \frac{1}{u} du = -\ln|1-x|$

$$INT = \lim_{t \rightarrow 1} \int_t^2 \left(\frac{1/2}{1-x} + \frac{1/2}{1+x} \right) dx$$

$$= \lim_{t \rightarrow 1} \frac{1}{2} \left[-\ln|1-x| + \ln|1+x| \right]_t^2$$

$$= \lim_{t \rightarrow 1} \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|_t^2$$

$$= \lim_{t \rightarrow 1} \frac{1}{2} \left[\ln(3) - \ln \left| \frac{1+t}{1-t} \right| \right]$$

$\rightarrow \ln|\infty| = \infty$ SINCE $\lim_{t \rightarrow 1} \frac{1+t}{1-t} \rightarrow \frac{2}{0} = \infty$

DIVERGES

2. The function $f(x) = \frac{1}{2}e^{3x}$ is a probability density function on the interval $[0, a]$. Find a .

$$\int_0^a \frac{1}{2}e^{3x} dx = 1$$

$$u = 3x$$

$$du = 3dx$$

$$\frac{1}{6} \int_0^{3a} e^u du = 1$$

$$\frac{1}{6} [e^{3a} - e^0] = 1$$

$$e^{3a} - 1 = 6$$

$$e^{3a} = 7$$

$$3a = \ln 7$$

$$a = \frac{1}{3} \ln 7$$