

Quiz 7

NAME: _____

RECITATION : Mon8 Mon9 Wed8 Wed9

1. Does the following series diverge or converge?

$$\sum_{n=1}^{\infty} 3^n \left[\sin \left(\frac{1}{n} \right) \right]^n$$

ROOT TEST:

$$\sqrt[n]{a_n} = \sqrt[n]{3^n \sin \left(\frac{1}{n} \right)^n} = 3 \sin \left(\frac{1}{n} \right)$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \sqrt[n]{a_n} &= \lim_{n \rightarrow \infty} 3 \sin \left(\frac{1}{n} \right) \\ &= 3 \sin \left(\lim_{n \rightarrow \infty} \frac{1}{n} \right) \quad \downarrow (f(x) = \sin x \text{ IS CONTINUOUS}) \\ &= 3 \sin 0 \\ &= 0 < 1 \end{aligned}$$

CONVERGES

2. Does the following series diverge or converge?

$$\sum_{n=1}^{\infty} \frac{3^n n^2}{(n!)^2}$$

RATIO TEST:

$$\begin{aligned} \frac{a_{n+1}}{a_n} &= \frac{3^{n+1} (n+1)^2}{(n+1)!^2} \cdot \frac{(n!)^2}{3^n n^2} \\ &= \frac{3(n+1)^2}{(n+1)^2 n^2} \\ &= \frac{3}{n^2} \rightarrow 0 < 1 \end{aligned}$$

CONVERGES