Quiz 8

NAME:______________________________

RECITATION : Mon8 Mon9 Wed8 Wed9

1. Find the radius and interval of convergence for the following series:

$$\sum_{n=1}^{\infty}(1+\tfrac{1}{n})^n x^n$$

ROOT TEST: $\sqrt[n]{|a_n|} = \sqrt[n]{(1+\frac{1}{n})^n |x|^n}$

$= (1+\frac{1}{n})|x|$

$\lim_{n\to\infty} \sqrt[n]{|a_n|} = |x| < 1 \quad (R=1)$

END PTS: $x=1$ THEN $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty}(1+\frac{1}{n})^n = e \neq 1$, DIV

$x=-1$ THEN $\sum(-1)^n(1+\frac{1}{n})^n$

AGAIN,

$\lim_{n\to\infty}|a_n| = \lim_{n\to\infty} a_n = e \neq 0$, DIV

$(-1, 1)$

(THIS PART WAS NOT NECESSARY TO SHOW)

RECALL $f(x) = (1+\frac{1}{x})^x$

$\ln f(x) = x\ln(1+\frac{1}{x})$

$\ln f(x) = \dfrac{\ln(1+\frac{1}{x})}{\frac{1}{x}}$

$\lim_{x\to\infty} \ln f(x) = \lim_{x\to\infty}\left[\dfrac{\frac{1}{(1+\frac{1}{x})}\left(\cancel{-\frac{1}{x^2}}\right)}{\cancel{-\frac{1}{x^2}}}\right]$

BY L'HOPITAL

$\lim_{x\to\infty} \ln f(x) = 1$

$\lim_{x\to\infty} f(x) = e^1 = e$

2. Find the interval of convergence for the following series, and find the sum of the series as a funtion of x:

$$\sum_{n=0}^{\infty} 3^n x^n$$

GEOMETRIC SERIES, $r = 3x$

$\downarrow$

CONV IF $|r| < 1$ $\quad |3x| < 1, \quad |x| < \frac{1}{3}$ CONV

DIV IF $|r| \geq 1$ $\quad |3x| \geq 1, \quad |x| \geq \frac{1}{3}$ DIV

$$\sum_{n \geq 0} (3x)^n = \frac{1}{1-3x} \quad \text{ON} \quad x \in \left(-\frac{1}{3}, \frac{1}{3}\right)$$