NOTE ON COMPLEX λ'S AND V'S:

WHEN SOLVING FOR COMPLEX EIGENVECTORS, SOMETIMES ROW REDUCTION OF THE SYSTEM $(A-\lambda I)v=0$ CAN BE MORE DIFFICULT DUE TO THE COMPLEX #'S. IN THE 2×2 CASE WE CAN GET AROUND THIS BY JUST PICKING A SOLUTION FOR ONE EQUATION AND VERIFY THAT IT WORKS FOR THE OTHER:

EX: $A = \begin{pmatrix} 3 & 2 \\ -2 & 3 \end{pmatrix}$ $\quad \begin{vmatrix} 3-\lambda & 2 \\ -2 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 + 4$

$$0 = \lambda^2 - 6\lambda + 13$$

$$\lambda = \frac{1}{2}\left(6 \pm \sqrt{36-4(13)}\right)$$

$$\lambda = 3 \pm 2i$$

LET $\lambda_1 = 3+2i$

$\lambda_2 = 3-2i$

TO FIND V_1:

$(A-\lambda_1 I)v = 0$

$$\left(\begin{array}{cc|c} 3-(3+2i) & 2 & 0 \\ -2 & 3-(3+2i) & 0 \end{array}\right)$$

IF WE FOUND THE CORRECT λ'S, THIS MATRIX IS RANK 1 SO THAT # L.I. λ's TO λ_1 = # PARAMS = $2 - \text{RK}(A-\lambda I) = 1$

IN OTHER WORDS, THE TWO EQUATIONS E_1 AND E_2 ARE JUST MULTIPLES OF ONE ANOTHER. THUS THEY SHOULD HAVE THE SAME SOLUTIONS.

$$\left(\begin{array}{cc|c} -2i & 2 & 0 \\ -2 & -2i & 0 \end{array}\right)$$

(IF $ax + by = 0$

$x = b,\ y = -a$ WORKS)

SO EQ$_1$: $-2ix + 2y = 0$

CHOOSE $X = 2$, $Y = 2i$ SO THAT THIS IS TRUE. THEN VERIFY THAT THIS SATISFIES EQ$_2$: $-2x - 2iy = 0$

$$-2(2) - 2i(2i) = 0$$

$$-4 - 4i^2 = 0$$

$$0 = 0 \checkmark$$

SO $V_1 = \begin{pmatrix} 2 \\ 2i \end{pmatrix}$

IN THIS CASE IT IS EASY TO SEE THAT $-iE_1 = E_2$ BUT IF SAY $(3-7i)E_1 = E_2$ IT WOULDN'T BE AS OBVIOUS.

THE POINT IS THAT YOU GET AROUND ROW REDUCTION BUT MAKE SURE TO CHECK THAT YOUR CHOSEN SOLUTION SATISFIES BOTH EQ'S.