Review Problems

#1 Let $A = \begin{pmatrix} 7 & 5 & 9 \\ 1 & -1 & 3 \\ 2 & 4 & 0 \end{pmatrix}$. Does the system of equations $Ax = b$ have a solution for any vector b?

#2 For what a and b does the matrix $A = \begin{pmatrix} a & -1 & 0 \\ b & 2 & -2 \\ 3-b & 0 & 4 \end{pmatrix}$ have an inverse?

#3 Compute the following obnoxious determinant:

$$\begin{vmatrix} 1 & 2 & 0 & -1 & 3 \\ -1 & -2 & 1 & 3 & 2 \\ 3 & 6 & 1 & 1 & -1 \\ 2 & 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & 1 & 3 \end{vmatrix}$$

#4 For what c is the following matrix diagonalizable?

$$\begin{pmatrix} 1 & 1+c \\ 1-c & 1 \end{pmatrix}$$

#5 If $A = \begin{pmatrix} 1 & 3 & 5 & -1 & 0 \\ 1 & 2 & 3 & 1 & 1 \\ 4 & 2 & 1 & -1 & 0 \end{pmatrix}$. Then the system of equations $Ax = 0$ has how many parameters?

#6 Find the general solution to the follow differential equation:

$$y'' - 2y' + 2y = e^x + 5$$

#7 Find the general solution to the following differential equation:

$$y'' + 2y' - 3y = 2e^{-3x}$$

#8 Solve the initial value problem:

$$x^2y'' - 2xy' - 4y = 0, \qquad y(1) = 0, y'(1) = 1$$

#9 (F03#3) Find the surface area of the piece of the paraboloid $z = 4 - x^2 - y^2$, $z \geq 1$.

#10 (S08#8) Let R be the region inside the surface $z^2 = x^2 + y^2$ from $z = 1$ to $z = 2$. Compute the flux integral $\int\int_{\partial R} F \cdot n dS$ where n is the outward normal and $F = (xy^2, x^2y, \sin x)$.

#11 (F05#3) Compute $\oint_C x dy$ where C is the triangular path traversing the points $(0,0) \mapsto (2,3) \mapsto (1,0) \mapsto (0,0)$ in that order.

#12 Let S be the portion of the sphere $x^2 + y^2 + (z-4)^2 = 25$ with $z \geq 0$. Compute the integral $\int\int_S \text{curl}F \cdot n dS$ where n is the usual outward pointing normal if we complete S to an entire sphere and $F = (y, y-x, z^2)$.

#13 Find two linearly independent series solution centered at $x = 0$ (say the first 4 nonzero terms of each) of the following differential equation:

$$(x^2 + 1)y'' - 6y = 0$$

#14 The position of a particle has a velocity vector $v = (\frac{dx}{dt}, \frac{dy}{dt})$ which satisfies the following differential equations:

$$\frac{dx}{dt} = -7x + 12y$$

$$\frac{dy}{dt} = -4x + 7y$$

If the particle starts at $\binom{1}{0}$ then where is it at time $t = 1$?