Math 361- Quiz 4

March 6, 2014

1. Consider the following region $E = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 1\}$. And consider the map $T : E \to \mathbb{R}^2$ defined as

$$T(x, y) = (x, (1 - x)y) = (u, v)$$

We will want to implement the Change of Variables Theorem in this problem

(a) (worth 2) Sketch $T(E)$

Proof. It is Q^2, a triangle in the first quadrant. □

(b) (worth 3) Show T is 1-1 in the appropriate region. State clearly what this region is that you are claiming T is 1-1 over. Also show that $J_T(x, y) \neq 0$ for all x in this region.

Proof. The region we want to show T is 1-1 over is *open*. Hence we pick

$$E' = \{(x, y) \in \mathbb{R}^2 : 0 < x < 1, 0 < y < 1\}$$

Now show T is 1-1 on E'. Also,

$$J_T(x, y) = 1 - x \neq 0 \qquad \forall x \in E'.$$

□

(c) (Worth 3) If we want to integrate a function $f : \mathbb{R}^2 \to \mathbb{R}$ over E what conditions must it satisfy so that we can use the Change of Variable Formula as given in class.

Proof. f must be continuous on $\mathbb{R}$, have compact support and $\text{supp}(f) \subseteq T(E')$. □

(d) (Worth 2) Give the Change of Variable formula.

Proof.

$$\int_{\mathbb{R}^2} f(u, v)dudv = \int_{\mathbb{R}^2} f(T(x, y))|J_T(x, y)|dxdy$$

□