Math 114-003 Fall 2013 Midterm 1

Exam B

October 29, 2013

Listed below are some practice T/F questions and answers to test your understanding of the material. Please note that actual T/F questions, while similar in style, will not necessarily be variants of the problems below. Also, showing work is neither required nor helpful on the actual exam.

1. **True** **False** Consider a differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ such that

$$f_x > 0,\ f_y < 0$$

everywhere. On each level set of f, the y coordinate must increase as the x coordinate increases.

True (On each level set of the form $f(x, y) = c$, $f_y \neq 0$ and hence $\partial y/\partial x = -f_x/f_y > 0$.)

2. **True** **False** For a differentiable function of the form

$$f : \mathbb{R}^3 \to \mathbb{R},$$

any point of f neither a local maximum nor a local minimum must be a saddle point.

False (A saddle point must also be a critical point.)

3. **True** **False** A critical point (x_0, y_0) of a differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ satisfying

$$f_{xx}(x_0, y_0) = f_{yy}(x_0, y_0) = 2f_{xy}(x_0, y_0)$$

is necessarily a local minimum.

False (It is possible for a function to have positive Hessian and positive f_{xx} while still satisfying the above conditions.)

4. **True** **False** Fix a differential function $f : \mathbb{R}^2 \to \mathbb{R}$ and $(x_0, y_0) \in \mathbb{R}^2$. There exists $C \in \mathbb{R}$ such that

$$|f(x, y) - L(x, y)| < C$$

for all x, y satisfying $(x - x_0)^2 + (y - y_0)^2 \leqslant 7$, where L is the standard linear approximation of f at (x_0, y_0).

True (The right side of the error term in the standard linear approximation is a continuous function on the closed and bounded region $(x - x_0)^2 + (y - y_0)^2 \leqslant 7$ and hence achieves a maximum C.)

5. **True** **False** A critical point (x_0, y_0) of a differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ satisfying

$$f_{xx}(x_0, y_0), f_{yy}(x_0, y_0) < 0$$

is necessarily a local maximum.

False (Such a condition is inconclusive if the Hessian is 0)

6. **True** **False** The triple integral $\int\int\int_D x^2 e^y z^4 \, dV$, where

$$D = \{(x, y, z) \mid z = x^2 + y^2\},$$

is positive.

False (The integral is 0. Every Reimann sum is 0 because no mesh cube lies inside D.)

7. **True** **False** The double integral $\int\int_D 16x^2 e^y \, dA$, where

$$D = \{(x, y) \mid x^2 + y^2 = 2\},$$

is positive.

False (Similar reason as above.)

8. **True** **False** For a differentiable function $f : \{(x, y, z) \in \mathbb{R}^3 \mid y > 0\} \to \mathbb{R}$ defined by

$$f(x, y, z) = e^x - \ln y + x^2 y z^2$$

a unit vector $\mathbf{u}$ maximizing $D_{\mathbf{u}} f$ at $(x_0, y_0, z_0) \in \mathbb{R}^3$ with $y_0 > 0$ is necessarily

$$\frac{1}{|(\nabla f)(x_0, y_0, z_0)|}(\nabla f)(x_0, y_0, z_0).$$

True (The gradient ∇f is defined whenever $y_0 > 0$. The desired $\mathbf{u}$ hence exists everywhere and is parallel to ∇f.)

9. **True** **False** For differentiable functions $f, g : \mathbb{R}^3 \to \mathbb{R}$,

$$\nabla fg = g\nabla f + f\nabla g.$$

True (Product rule for gradients)

10. **True** **False** For a function $f : \mathbb{R}^2 \to \mathbb{R}$ with f_x, f_y, f_{xy}, f_{yx} defined and continuous everywhere,

$$f_{xy} = f_{yx}$$

is necessarily true.

True (Clairaut's Mixed Derivative Theorem)

11. **True** **False** For a differential function $f : \mathbb{R}^2 \to \mathbb{R}$, the difference

$$|f(x_0, y_0) - L(x_0, y_0)|,$$

where L is the standard linear approximation of f at (x_0, y_0), is not necessarily 0.

False ($L(x_0, y_0) = f(x_0, y_0)$)

12. **True** **False** A continuous function $f : D \to \mathbb{R}$ must have an absolute minimum if

$$D = \{(x, y, z) \mid |x| + |y| + |z| \leqslant 5\}.$$

True (The domain D is closed and bounded, so f achieves its extremum by the Extreme Value Theorem.)