

Practice Problems for Midterm 2

March 26, 2008

1. In a lottery, the players select 6 different numbers between 1 and 15. What is the probability that the winning combination consists of all even numbers?
2. A deck of 52 cards is distributed evenly among 4 people. What is the probability that one person receives exactly 3 aces?
3. If you put 6 pairs of socks in the laundry, each of a different color, and withdraw 9 individual socks, what is the expected number of pairs that you withdraw?
4. There are 150 students in a class, 30 speak Chinese, 50 speak Spanish, 75 speak French. Twenty percent of the Spanish speaking students speak all three languages. The probability that a French speaking student also speaks Spanish is $\frac{4}{25}$, the probability that a Chinese speaking student also speaks French is $\frac{1}{2}$, and the probability that a Spanish speaking student speaks Chinese is $\frac{9}{25}$. If you choose a student at random from the class, what is the probability that he or she speaks none of these three languages?
5. 6% of Type A spark plugs are defective, 4% of Type B spark plugs are defective and 2% of Type C spark plugs are defective. A spark plug is selected at random from a batch of 80 Type A plugs, 40 Type B plugs and 20 Type C plugs. The selected plug is found to be defective. What is the probability that the selected plug was of Type A?
6. John and Karen play 5 games of chess and it is decided after a 5 game championship that the person who has won the most games is the champion. John has a 40% chance of winning any particular game against Karen. It is known that the winner won 3 of the five games. What is the probability that the winner was John?
7. X and Y are independent exponentially distributed random variables with mean waiting time 3 and 4 hours respectively (i.e. $f(x) = \frac{1}{3}e^{-\frac{x}{3}}$, $f(y) = \frac{1}{4}e^{-\frac{y}{4}}$). What is the probability that event X occurs before event Y given that you have already waited 1 hour and neither event has occurred?
8. X is a continuous random variable on $[0, 1]$ whose density function is of the form $f(x) = kx^2$. What is $Var(X)$?

9. A point (x, y) is chosen randomly on a rectangle of 3 by 5 feet. What is the probability that $|x - y| > 2$?
10. Suppose that a random variable X is uniformly distributed on the interval $[1, 6]$. The expected value of $\frac{1}{X+1}$ is:
11. Three dice are rolled and the sum is found to be 7. What is the probability that none of three faces showing was a two?
12. Let the bivariate distribution of two continuous variables, X and Y , be given by

$$f(x, y) = c(x + y), \quad 0 \leq x \leq 2, 0 \leq y \leq 1 \\ = 0 \text{ otherwise.}$$

Calculate $P(X > Y | X < 1)$.

13. You have four friends and each of them has 4 coins, 1 cent, 5 cents, 10 cents and 25 cents. Each of your friends gives you a coin at random. Compute the expected value of the money you receive.
14. Suppose a particle starts at the origin of the real line and moves along the line at jumps of one unit. For each jump, the probability is $2/3$ that the particle jumps to the left and the probability is $1/3$ that the particle moves to the right. What is the expected position of the particle after 10 jumps?

Answers

1. $\frac{1}{55 \times 13}$
2. $\frac{\binom{4}{1}\binom{13}{3}\binom{49}{1}}{\binom{52}{4}}$
3. $\frac{36}{11}$
4. $\frac{1}{5}$
5. $\frac{12}{17}$
6. .4
7. $\frac{3}{7}$
8. $\frac{3}{80}$
9. $\frac{1}{3}$
10. $\ln(7/2)$
11. .4
12. $\frac{1}{2}$
13. \$.41
14. $-\frac{10}{3}$