Homework for Sections 16.1–16.3

Mathematics 114, Section 2

due Wednesday, November 27

Read the green pamphlet "Preview of Differential Equations," Sections 16.1–16.3.

1. (16.1, #6) Show that each function $y = f(x)$ is a solution of the differential equation.

$$y' + \frac{1}{x}\,y = 1$$

$$\text{a) } y = \frac{x}{2} \qquad \text{b) } y = \frac{1}{x} + \frac{x}{2} \qquad \text{c) } y = \frac{C}{x} + \frac{x}{2}$$

2. (16.1, #10) Show that the function $y = f(x)$ is a solution of the given differential equation.

$$y = \frac{1}{x}\int_1^x \frac{e^t}{t}\,dt, \qquad x^2 y' + xy = e^x$$

3. (16.1, #15) Solve the initial value problem.

$$\frac{dy}{dx} = \frac{2y^2 + 2}{x^2 - 1}, \qquad y(0) = 1$$

4. (16.1, #24) Show that the equation is homogeneous and find its general solution.

$$(x^2 + y^2)\,dx + xy\,dy = 0$$

5. (16.2, #4) Test the equation for exactness.

$$(x + y^2)\,dx + (2xy + 1)\,dy = 0$$

6. (16.2, #9) Solve the equation.

$$(x + y)\,dx + (x + y^2)\,dy = 0$$

7. (16.2, #14) Solve the equation.

$$\frac{dy}{dx} = \frac{y \cos xy + 1}{3 - 2y - x \cos xy}$$

8. (16.2, #19) Show that the integrating factor ρ makes the differential equation exact. Then solve the equation using that integrating factor.

$$(xy^2 + y)\,dx - x\,dy = 0; \qquad \rho = 1/y^2$$

9. (16.2, #26) Solve the initial value problem for y as a function of x.

$$(2x + y + 1)\,dx + (2y + x + 1)\,dy = 0, \qquad y(1) = 5$$

10. (16.3, #4) Solve the differential equation.

$$x\,\frac{dy}{dx} + y = x \cos x$$

11. (16.3, #7) Solve the differential equation.

$$\sin x\,\frac{dy}{dx} + (\cos x)y = \tan x$$

12. (16.3, #9) Solve the initial value problem for y as a function of x.

$$\frac{dy}{dx} + 2y = x, \qquad y(0) = 1$$

13. (16.3, #12) Solve the initial value problem for y as a function of x.

$$x\,\frac{dy}{dx} - 2y = x^3 \sec x \tan x, \qquad y(\pi/3) = 2$$

14. (16.3, #18) *Continuous compounding.* You have \$1000 with which to open an account and plan to add \$1000 per year. All funds in the account will earn 10% interest per year compounded continuously. If the added deposits are also credited to your account continuously, the number of dollars x in your account at time t (years) will satisfy the initial value problem

$$\frac{dx}{dt} = 1000 + 0.10x, \qquad x(0) = 1000$$

a) Solve the initial value problem for x as a function of t.

b) About how many years will it take for the amount in your account to reach \$100,000?

Make sure you can do the core problems.
(16.2 — 5,13,20; 16.3 — 1,3,10,11,19).