Brief answers to Homework #9

Section 12.9, #8 Find the extrema of the squared distance $f(x, y) = x^2 + y^2$ with respect to the constraint $g(x, y) = x^2 + xy + y^2 = 1$.

The points closest to the origin are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ and $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$, while the ones farthest from the origin are $(1, -1)$ and $(-1, 1)$.

Section 12.9, #11 Maximize $f(x, y) = 4xy$ with constraint $g(x, y) = x^2/16 + y^2/9 = 1$. The maximum occurs at $x = 2\sqrt{2}$, $y = \frac{3\sqrt{2}}{2}$.

Section 12.9, #18 Maximize $f(x, y, z) = (x-1)^2 + (y+1)^2 + (z-1)^2$ with respect to the constraint $g(x, y, z) = x^2 + y^2 + z^2 = 4$. The farthest point is $\left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right)$.

Section 12.9, #25 Minimize $f(x, y, z) = x^2 + y^2 + z^2$ with respect to the constraint $g(x, y, z) = x + y + z = 9$. The three numbers are $3, 3, 3$.

Section 12.9, #26 Maximize $f(x, y, z) = xyz$ subject to the constraint $g(x, y, z) = x + y + z^2 = 16$. The maximum occurs when $x = \frac{32}{5}$, $y = \frac{32}{5}$, and $z = \frac{4}{\sqrt{5}}$. The maximum value is $\frac{4096}{25\sqrt{5}}$.

Section 12.9, #42 See "least squares plane" writeup on web page.

Section 13.1, #3

$$\int_{-1}^{0} \int_{-1}^{1} (x + y + 1)\, dx\, dy = \int_{-1}^{0} (2 + 2y)\, dy = 1$$

Section 13.1, #6

$$\int_0^{\pi}\int_0^{\sin x} y\,dy\,dx = \int_0^{\pi} \tfrac{1}{2}\sin^2 x\,dx = \frac{\pi}{4}$$

Section 13.1, #11

$$\int_1^2\int_x^{2x} \frac{x}{y}\,dy\,dx = \int_1^2 (\ln 2)x\,dx = \frac{3}{2}\ln 2$$

Section 13.1, #14

$$\int_0^1\int_0^{\pi} y\cos xy\,dx\,dy = \int_0^1 \sin \pi x\,dx = \frac{2}{\pi}$$

Section 13.1, #22

$$\int_0^2\int_{y-2}^{0} dx\,dy = \int_{-2}^{0}\int_0^{x+2} dy\,dx$$

Section 13.1, #25

$$\int_0^1\int_1^{e^x} dy\,dx = \int_1^{e}\int_{\ln y}^{1} dx\,dy$$

Section 13.1, #28

$$\int_0^2\int_0^{4-y^2} y\,dx\,dy = \int_0^4\int_0^{\sqrt{4-x}} y\,dy\,dx$$

Section 13.1, #35

$$\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\, dx\, dy = \int_0^{\sqrt{\ln 3}} \int_0^{2x} e^{x^2}\, dy\, dx = 2$$

Section 13.1, #46

$$V = \int_0^2 \int_0^{4-x^2} (4 - x^2 - y)\, dy\, dx = \frac{128}{15}$$

Section 13.1, #51

$$\int_1^{\infty} \int_{e^{-x}}^{1} \frac{1}{x^3 y}\, dy\, dx = \int_1^{\infty} \frac{1}{x^2}\, dx = 1$$

Section 13.1, #54

$$\int_0^{\infty} \int_0^{\infty} x e^{-(x+2y)}\, dx\, dy = \int_0^{\infty} e^{-2y}\, dy = \frac{1}{2}$$