

Introduction to mathematical logic: Homework

Mathematics 170

due Tuesday, Feb. 4

1. Let G = “Al Gore will run for President,” L = “Joe Lieberman will run for President,” and K = “John Kerry will run for President.” Write the following sentences symbolically.
 - (a) If Gore runs for President, Lieberman will not.
 - (b) Kerry and Gore will not both run.
 - (c) If neither Kerry nor Lieberman run, Gore will.
2. Let Q = “The U.S. will go to war with Iraq,” let N = “The U.S. will go to war with Iran,” and let K = “The U.S. will go to war with North Korea.” Write the following symbolic statements as English sentences.
 - (a) $Q \Rightarrow (N \vee K)$.
 - (b) $(Q \vee N) \wedge \neg(Q \wedge N)$.
 - (c) $\neg(Q \wedge \neg K)$.
3. Construct a truth table for the statement $\neg(P \vee Q) \wedge R$.
4. Show that the statements $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$ are equivalent, by showing that they have the same truth table.
5. Show that the statements $(P \wedge Q) \vee R$ and $P \wedge (Q \vee R)$ are not equivalent, by showing their truth tables are not the same.
6. Prove using a truth table that a theorem is always equivalent to its contrapositive; that is, show that $P \Rightarrow Q$ has the same truth table as $\neg Q \Rightarrow \neg P$.

7. How many different truth tables could be constructed for an operation between two statements P and Q ?
8. Write an if-then statement about the President whose converse is true. Write one about the Vice President whose converse is false.
9. Suppose “If taxes are cut for the rich, the recession will end.” is a true statement. Which of the following must be true statements?
 - (a) “If taxes are not cut for the rich, the recession will not end.”
 - (b) “If the recession doesn’t end, taxes for the rich haven’t been cut.”
 - (c) “If the recession ends, taxes have been cut for the rich.”
10. A certain bill came up for a vote in the Senate. If Trent Lott voted for it, Ted Kennedy promised not to vote for it. Jim Jeffords said he’d vote for it only if Lott voted for it. If Kennedy voted for it, then either Jeffords or Lott voted against it.
 - (a) Let J , K , and L denote the statements “Jeffords voted yes,” “Kennedy voted yes,” and “Lott voted yes,” respectively. Assuming everyone kept their promises, write the assumptions above symbolically.
 - (b) If Kennedy voted for the bill, determine how Jeffords and Lott voted. Give symbolic arguments for each deduction, using the axioms you wrote in part (a).