# O'Neill's Theorem: Homework

Mathematics 170

due Tuesday, Feb. 25

1. *Mathematics and Politics*, pg. 19, problems 15 and 16.

2. *Mathematics and Politics*, pg. 152, problem 9.

3. Consider the dollar auction with the rule that the player who loses pays *half* his/her last bid, while the player who wins pays all of his/her bid and gets \$1.00. Suppose the bankroll is \$5.00 and the possible bid increments are \$0.10, and that both players use the conservative convention. Use the technique discussed in class (and Section 6.2) to find the optimal opening bid. Here are some hints.

   (a) Explain why, if player two has bid \$1.70 or more, you cannot lose more than \$0.80 by passing and cannot gain more than \$0.80 by bidding higher. What should you do?

   (b) If player two has bid \$1.60 you might gain by bidding \$1.70. Under what circumstances is it better to bid higher? What should you bid?

   (c) Show that you should always pass if player two bids \$1.50.

   (d) If player two has bid between \$1.20 and \$1.40, you could win by bidding \$1.50. Under what circumstances should you do this?

   (e) Show that you should always pass if player two bids \$1.10.

   (f) If player two bids less than \$1.10, you can win by bidding \$1.10. Under what circumstances should you do this?

   (g) What is the next lowest bid which forces the other player to pass? Explain why this is the optimal opening bid.

4. What features of the dollar auction are relevant to understanding real international conflict? Which features are not relevant? What does O'Neill's Theorem tell us about strategies in real international conflict? Give your opinion in no more than two paragraphs.